The formulas in the box will be provided on the exam.

$$\kappa(s) = \left|\left|\frac{d\mathbf{T}}{ds}\right|\right| \qquad \kappa(x) = \frac{|f''(x)|}{[1+(f'(x))^2]^{3/2}}$$

$$\kappa(t) = \frac{||\mathbf{T}'(t)||}{||\mathbf{r}'(t)||} \qquad \kappa(t) = \frac{||\mathbf{r}'(t) \times \mathbf{r}''(t)||}{||\mathbf{r}'(t)||^3}$$

1. True or False? Circle ONE answer for each. *Hint: For effective study, explain why if 'true' and give a counterexample if 'false.'*
   (a) T or F: If $\mathbf{a} \perp \mathbf{b}$ and $\mathbf{b} \perp \mathbf{c}$, then $\mathbf{a} \perp \mathbf{c}$.
   (b) T or F: If $\mathbf{a} \cdot \mathbf{b} = 0$, then $||\mathbf{a} \times \mathbf{b}|| = ||\mathbf{a}||||\mathbf{b}||$.
   (c) T or F: For any vectors $\mathbf{u}, \mathbf{v}$ in $\mathbb{R}^3$, $||\mathbf{u} \times \mathbf{v}|| = ||\mathbf{v} \times \mathbf{u}||$.
   (d) T or F: The vector $< 3, -1, 2 >$ is parallel to the plane $6x - 2y + 4z = 1$.
   (e) T or F: If $\mathbf{u} \cdot \mathbf{v} = 0$, then $\mathbf{u} = \mathbf{0}$ or $\mathbf{v} = \mathbf{0}$.
   (f) T or F: If $\mathbf{u} \times \mathbf{v} = \mathbf{0}$, then $\mathbf{u} = \mathbf{0}$ or $\mathbf{v} = \mathbf{0}$.
   (g) T or F: If $\mathbf{u} \cdot \mathbf{v} = 0$ and $\mathbf{u} \times \mathbf{v} = \mathbf{0}$, then $\mathbf{u} = \mathbf{0}$ or $\mathbf{v} = \mathbf{0}$.
   (h) T or F: The curve $\mathbf{r}(t) = \langle 0, t^2, 4t \rangle$ is a parabola.
   (i) T or F: If $\kappa(t) = 0$ for all t, the curve is a straight line.
   (j) T or F: Different parameterizations of the same curve result in identical tangent vectors at a given point on the curve.

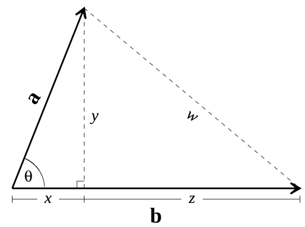
2. Which of the following are vectors?

(a)	○ Vector	○ Scalar	○ Nonsense	$[(\mathbf{a} \cdot \mathbf{b})\mathbf{c}] \times \mathbf{a}$
(b)	○ Vector	○ Scalar	○ Nonsense	$\mathbf{c} \times [(\mathbf{a} \cdot \mathbf{b}) \times \mathbf{c}]$
(c)	○ Vector	○ Scalar	○ Nonsense	$\mathbf{c} \times [(\mathbf{a} \cdot \mathbf{b})\mathbf{c}]$
(d)	○ Vector	○ Scalar	○ Nonsense	$(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$

3. Which of the following are meaningful?

(a)	○ Meaningful	○ Nonsense	$||\mathbf{w}||(\mathbf{u} \times \mathbf{v})$
(b)	○ Meaningful	○ Nonsense	$(\mathbf{u} \cdot \mathbf{v}) \times \mathbf{w}$
(c)	○ Meaningful	○ Nonsense	$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$

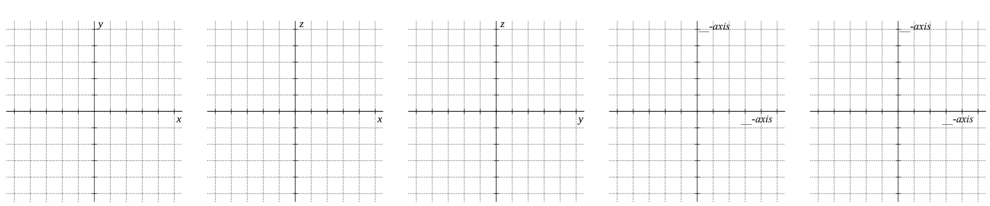
4. Find the values of x such that the vectors $< 3, 2, x >$ and $< 2x, 4, x >$ are orthogonal.
5. Find the decomposition $\mathbf{a} = \mathbf{a}_{||\mathbf{b}} + \mathbf{a}_{\perp\mathbf{b}}$ of $\mathbf{a} =< 1, 1, 1 >$ along $\mathbf{b} =< 2, -1, -3 >$.
6. Let $\mathbf{a} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle, \mathbf{b} = \left\langle \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}} \right\rangle$, and $\mathbf{u} =< 3, 0 >$.
   (a) Show that $\mathbf{a}$ and $\mathbf{b}$ are orthogonal unit vectors.
   (b) Find the decomposition of $\mathbf{u}$ along $\mathbf{a}$.
   (c) Find the decomposition of $\mathbf{u}$ along $\mathbf{b}$.
7. (a) Find an equation of the sphere that passes through the point (6, -2, 3) and has center (-1,2,1).
   (b) Find the curve in which this sphere intersects the yz-plane.
8. For each of the following quantities ($\cos\theta, \sin\theta, x, y, z$, and w) in the picture below, fill in the blank with the number of the expression, taken from the list to the right, to which it is equal.



$\cos\theta =$ ________________	1. $\dfrac{\mathbf{a}\cdot\mathbf{b}}{\|\|\mathbf{a}\|\|}$
$\sin\theta=$ ________________	2. $\dfrac{\mathbf{a}\cdot\mathbf{b}}{\|\|\mathbf{b}\|\|}$
$x =$ ________________	3. $\dfrac{\|\|\mathbf{a}\times\mathbf{b}\|\|}{\|\|\mathbf{b}\|\|}$
$y=$ ________________	4. $\dfrac{\|\|\mathbf{a}\times\mathbf{b}\|\|}{\|\|\mathbf{a}\|\|\|\|\mathbf{b}\|\|}$
$z=$ ________________	5. $\dfrac{\|\mathbf{a}\cdot\mathbf{b}\|}{\|\|\mathbf{a}\|\|\|\|\mathbf{b}\|\|}$
$w=$ ________________	6. $\|\|\mathbf{b}-\mathbf{a}\|\|$
	7. $\dfrac{(\mathbf{b}-\mathbf{a})\cdot\mathbf{b}}{\|\|\mathbf{b}\|\|}$

9. Find an equation for the line through $(4,-1,2)$ and $(1,1,5)$.
10. Find an equation for the line through $(-2,2,4)$ and perpendicular to the plane $2x-y+5z=12$.
11. Find an equation of the plane through $(2,1,0)$ and parallel to $x+4y-3z=1$.
12. Find an equation of the plane that passes through the point $(-1,-3,2)$ and contains the line $x(t) = -1-2t,\ y(t)=4t,\ z(t)=2+t$.
13. Find the point at which the line $x(t)=1-t,\ y=t,\ z(t)=1+t$ and the plane $z=1-2x+y$ intersect.
14. (a) Find an equation of the plane that passes through the points $A(2,1,1)$, $B(-1,-1,10)$, and $C(1,3,-4.)$
    (b) A second plane passes through $(2,0,4)$ and has normal vector $<2,-4,-3>$. Find an equation for the line of intersection of the two planes.
15. Provide a clear sketch of the following traces for the quadratic surface $y=\sqrt{x^2+z^2}+1$ in the given planes. Label your work appropriately.

$$x=0; x=1; y=0; y=2; z=0.$$

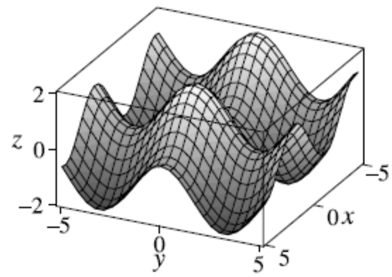


16. Match the equations with their graphs. Give reasons for your choices.
    (a) ________________ $8x+2y+3z=0$
    (b) ________________ $z=\sin x+\cos y$

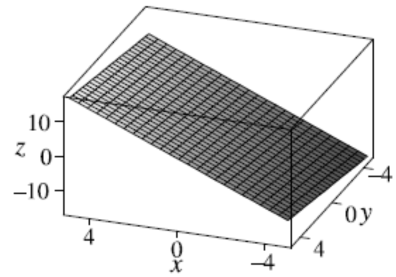
(c) ______________ $z = \sin\left(\dfrac{\pi}{2 + x^2 + y^2}\right)$

(d) ______________ $z = e^y$

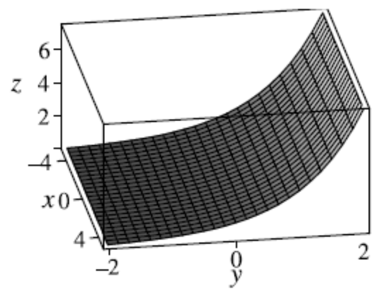
I



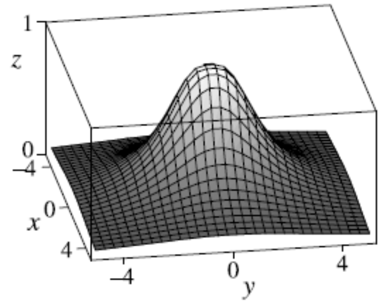
II



III



IV



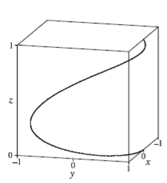
17. Find a vector function that represents the curve of intersection of the cylinder $x^2 + y^2 = 16$ and the plane $x + z = 5$.

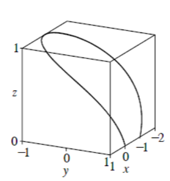
18. Find an equation for the tangent line to the curve $x = 2\sin t$, $y = 2\sin 2t$, and $z = 2\sin 3t$ at the point $(1, \sqrt{3}, 2)$.

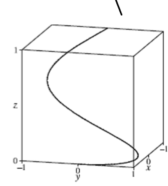
19. A helix circles the $z-$axis, going from $(2, 0, 0)$ to $(2, 0, 6\pi)$ in one turn.
   (a) Parameterize this helix.
   (b) Calculate the length of a single turn.
   (c) Find the curvature of this helix.

20. (a) Sketch the curve with vector function $\mathbf{r}(t) = \langle t, \cos \pi t, \sin \pi t\rangle, t \geq 0$.
   (b) Find $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$.

21. Which curve below is traced out by $\mathbf{r}(t) = \left\langle \sin \pi t, \cos \pi t, \dfrac{1}{4}t^2 \right\rangle$, $0 \leq t \leq 2$.


Graph 1


Graph 2


Graph 3

22. Find a point on the curve $\mathbf{r}(t) = \langle t + 1, 2t^2 - 2, 5\rangle$ where the tangent line is parallel to the plane $x + 2y - 4z = 5$.

23. Let $\mathbf{r}(t) = \langle \sqrt{2 - t}, (e^t - 1)/t, \ln(t + 1)\rangle$.
   (a) Find the domain of $\mathbf{r}$.
   (b) Find $\lim\limits_{t\to 0} \mathbf{r}(t)$.
   (c) Find $\mathbf{r}'(t)$.

24. Suppose that an object has velocity $\mathbf{v}(t) = \langle 3\sqrt{1 + t}, 2\sin(2t), 6e^{3t}\rangle$ at time t, and position $\mathbf{r}(t) =< 0, 1, 2 >$ at time $t = 0$. Find the position, $\mathbf{r}(t)$, of the object at time t.

25. If $\mathbf{r}(t) = \langle t^2, t\cos\pi t, \sin\pi t\rangle$, evaluate $\int_0^1 \mathbf{r}(t)dt$.

26. Find the length of the curve: $x = 2\cos(2t)$, $y = 2t^{3/2}$, and $z = 2\sin(2t)$; $0 \le t \le 1$.

27. Reparameterize the curve $\mathbf{r}(t) =< e^t, e^t \sin t, e^t \cos t >$ with respect to arc length measured from the point $(1, 0, 1)$ in the direction of increasing t.

28. Find the tangent line to the curve of intersection of the cylinder $x^2 + y^2 = 25$ and the plane $x = z$ at the point $(3, 4, 3)$.

29. For the curve given by $\mathbf{r}(t) = \langle \frac{1}{3}t^3, t^2, 2t\rangle$, find
    (a) the unit tangent vector
    (b) the unit normal vector
    (c) the curvature

30. A particle moves with position function $\mathbf{r}(t) =< t\ln t, t, e^{-t} >$. Find the velocity, speed, and acceleration of the particle.

31. A particle starts at the origin with initial velocity $< 1, -1, 3 >$ and its acceleration is $\mathbf{a}(t) =< 6t, 12t^2, -6t >$. Find its position function.

32. A flying squirrel has position $\mathbf{r}(\mathbf{t}) = \left\langle t + \frac{t^2}{2}, 1 - t, 2 + t^2 \right\rangle$ at time t. Compute the following at time $t = 1$:
    (a) The velocity at time $t = 1$, $\mathbf{v}(1) = < 2, -1, 2 >$.
    (b) The speed at time $t = 1$, $\nu(1) =$ ______________.

33. Consider the vector valued function $\mathbf{r}(t) =$ describing the curve shown below. Put the curvature of $\mathbf{r}$ at A, B and C in order from smallest to largest. Draw the osculating circles at those points.