and 15.6

M273 Exam 3 Review Problems

Exam 3 covers material from Sections 15.1-15.4 and 16.1-16.3 and 17.1.

The formulas in the box will be provided on the exam.

$dA = r\,dr\,d\theta$	$dV = r\,dz\,dr\,d\theta$	$dV = \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta$

1. Let $\mathbf{F}$ be a vector field in $\mathbb{R}^3$ and f a scalar function of three variables. For each of the following, state whether the operations shown produce a vector field, a scalar function, or whether they cannot be computed, in which case the statement is nonsense.

   ○ Vector ○ Scalar ○ Nonsense $\nabla \cdot \mathbf{F}$
   ○ Vector ○ Scalar ○ Nonsense $\nabla(\nabla \cdot \mathbf{F})$
   ○ Vector ○ Scalar ○ Nonsense $\nabla \times (\nabla \times \mathbf{F})$
   ○ Vector ○ Scalar ○ Nonsense $(\nabla f) \times \mathbf{F}$
   ○ Vector ○ Scalar ○ Nonsense $(\nabla \cdot \mathbf{F}) \cdot \mathbf{F}$

2. Calculate $\displaystyle\iint_{\mathcal{D}} x\,dA$ where $\mathcal{D}$ is the triangular region with vertices $(0,0)$, $(1,1)$, and $(1,-1)$.

3. Evaluate $\displaystyle\int_1^e \int_0^{1/x} xye^{xy}\,dy\,dx$.

4. Evaluate $\displaystyle\int_0^{2\sqrt{\pi}} \int_{x/2}^{\sqrt{\pi}} \sin y^2\,dy\,dx$.

5. Let $\mathcal{W} = \left\{(x,y,z) | x^2+y^2+z^2 \le 4, 0 \le x \le \sqrt{3}, y \ge 0\,, z \ge 0\right\}$. Evaluate $\displaystyle\iiint_{\mathcal{W}} xz\,dV$.

6. Calculate $\displaystyle\int_0^1 \int_y^{\sqrt{2-y^2}} x^2+y^2\,dx\,dy$ by changing to polar coordinates.

7. (a) Sketch the region of integration for $\displaystyle\int_{-1}^1 \int_{x^2}^1 \left(1+y^{3/2}\right)^5 dy\,dx$.

   (b) Switch the order of integration for the integral in part a.

   (c) Calculate the integral from part b.

8. Set up (**but do not evaluate**) iterated triple integrals, with appropriate limits, for find the volume of the solid bounded by $z = x^2+y^2$ and $z = 8-x^2-y^2$ in:

   (a) rectangular coordinates

   (b) cylindrical coordinates

9. Set up an iterated triple integral, with the appropriate limits of integration, in the coordinate system of your choice, for finding the volume of the region that lies between the spheres $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=9$, and "inside" the cone $z=\sqrt{x^2+y^2}$. (That is, $z \ge \sqrt{x^2+y^2}$). **DO NOT EVALUATE.**

10. Convert $\displaystyle\int_0^4 \int_{-\sqrt{4-x}}^{\sqrt{4-x}} \int_0^{1-x/4} f(x,y,z)\,dz\,dy\,dx$ to an integral in $dy\,dx\,dz$ order.

11. Express, as an iterated triple integral, the volume of the solid given by $x^2+y^2 \le z \le 2x+4y+4$.

12. Circle whether the following quantities are vectors, scalars, or are nonsensical (that is, the statement is not defined or does not make sense):

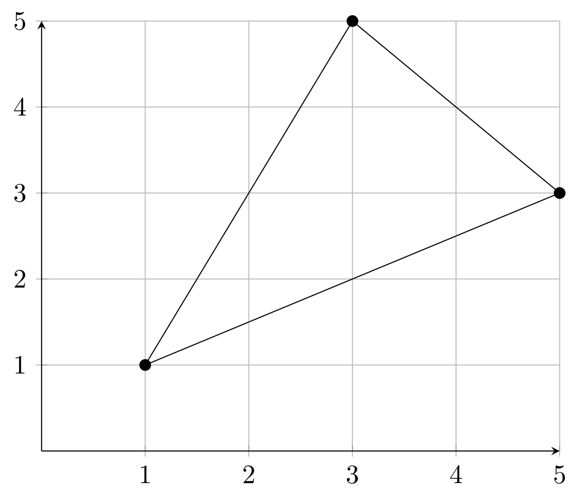
◯ Vector	◯ Scalar	◯ Nonsense	∇f
◯ Vector	◯ Scalar	◯ Nonsense	$\nabla \mathbf{F}$
◯ Vector	◯ Scalar	◯ Nonsense	$\nabla \times \mathbf{F}$
◯ Vector	◯ Scalar	◯ Nonsense	$\nabla(\nabla \times \mathbf{F})$
◯ Vector	◯ Scalar	◯ Nonsense	$\nabla \cdot f$

13. Sketch the domain of $\int_0^1 \int_{e^x}^{e} f(x,y)\,dy\,dx$ and then express it as an iterated integral in the opposite order. Do not evaluate the resulting integral.

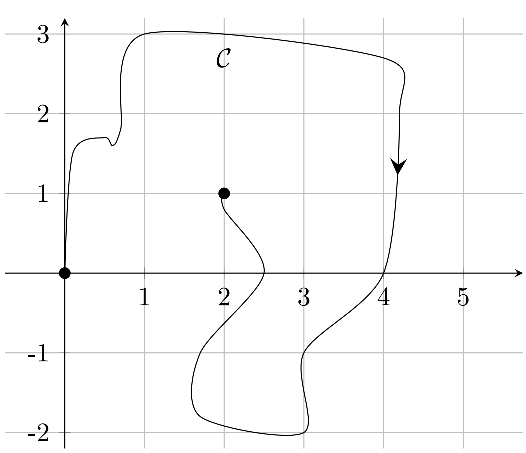
14. Evaluate $\iint_{\mathcal{D}} y(1+x^2)^{-1} dA$, where $\mathcal{D}$ is the region bounded by $y=\sqrt{x}$, $x=1$ and the $x-$axis.

15. Evaluate $\iint_{\mathcal{D}} e^{y^2} dA$, where $\mathcal{D}$ is the region bounded by $\frac{x}{2} \le y \le 2$ and $0 \le x \le 4$.

16. Evaluate the volume of the solid under $f(x,y) = x+1$ over the triangle with vertices $(1,1), (5,3), (3,5)$, pictured right.



17. Calculate the following line integrals
   (a) $\int_{\mathcal{C}} (x+y)\,ds$ where C is the straight line segment from $(1,2)$ to $(4,6)$.
   (b) $\int_{\mathcal{C}} x\,dy - y\,dx$ where C is the semicircle $x^2 + y^2 = 4$ where $y \ge 0$ and is oriented counterclockwise.
   (c) $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x,y) = \langle 2x+y, x-2y \rangle$ and C is as pictured below.



18. Evaluate $\int_{\mathcal{C}} ds$ where the curve $\mathcal{C}$ is parametrized by $\mathbf{r}(t) = \langle 4t, -3t, 12t \rangle$ for $2 \le t \le 5$. What does this integral represent?

19. Calculate the total mass of a metal tube in the helical shape $\mathbf{r}(t) = \langle \cos(t), \sin(t), t^2 \rangle$ (distance in centimeters) for $0 \le t \le 2\pi$ if the mass density is $\delta(x,y,z) = \sqrt{z}$ g/cm.

20. Find the work done by the force field $\mathbf{F} = \langle x^2, y^2, z^2 \rangle$ in moving a particle along the piecewise linear path from $(0,0,0)$ to $(1,2,-1)$ and then from $(1,2,-1)$ to $(3,2,0)$.

21. Let $\mathbf{F} = \langle x\sin(2x), e^y + e^{-z}, e^y - e^{-z}\rangle$.
    (a) Show that $\mathbf{F}$ is not conservative.
    (b) Compute $\nabla \cdot \mathbf{F}$.
    (c) Compute $\nabla \times \mathbf{F}$.

22. Let $d = \sqrt{x^2+y^2+z^2}$ and $\mathbf{F} = \dfrac{1}{d^2}\langle x, y, z\rangle$.
    (a) Show that $f(x, y, z) = \ln(d)$ is a potential for $\mathbf{F}$.
    (b) Find the divergence of $\mathbf{F}$.
    (c) Verify that the curl$(\mathbf{F}) = \mathbf{0}$.

23. Let $\mathbf{F}(x, y, z)$ be a vector field and $\mathbf{r}(t)$ for $a \le t \le b$ be a parameterization of a curve $\mathcal{C}$. Circle the correct completion of each sentence.

    ○ Vector Line Integral ○ Scalar Line Integral ○ Nonsense $\displaystyle\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$

    ○ Vector Line Integral ○ Scalar Line Integral ○ Nonsense $\displaystyle\int_{\mathcal{C}} (\nabla \cdot \mathbf{F})\, d\mathbf{r}$

    ○ Vector Line Integral ○ Scalar Line Integral ○ Nonsense $\displaystyle\int_{\mathcal{C}} (\nabla \cdot \mathbf{F})\, ds$

    ○ Vector Line Integral ○ Scalar Line Integral ○ Nonsense $\displaystyle\int_{\mathcal{C}} (\nabla \times \mathbf{F}) \cdot d\mathbf{r}$

24. (a) Compute $\displaystyle\int_{\mathcal{C}} \langle y, x\rangle \cdot d\mathbf{r}$ where $\mathcal{C}$ is three quarters of the circle of radius $\sqrt{2}$ oriented clockwise from $(1, -1)$ to $(1, 1)$.
    (b) Compute $\displaystyle\int_{\mathcal{C}} \langle -y, x\rangle \cdot d\mathbf{r}$ where $\mathcal{C}$ is three quarters of the circle of radius $\sqrt{2}$ oriented clockwise from $(1, -1)$ to $(1, 1)$.

25. Let $f(x, y) = e^{x^2+y^2}$. Compute $\displaystyle\int_{\mathcal{C}} \nabla f \cdot d\mathbf{r}$ where $\mathcal{C}$ is the line segment from $(-1, 0)$ to $(3, 3)$.

26. Let $\mathbf{F}(x, y) = \left\langle \dfrac{y}{1+(xy)^2}, \dfrac{x}{1+(xy)^2} \right\rangle$.
    (a) Find a potential function for $\mathbf{F}$.
    (b) Calculate $\displaystyle\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$ where $\mathcal{C}$ is any curve from the origin to (a, a^{-1}) and a is any nonzero constant.

27. Let $\mathbf{F}(x, y, z) = \langle 2x + 3y, 2y, 2z\rangle$.
    (a) Show that $\mathbf{F}$ is not conservative.
    (b) Calculate the work done by the vector field $\mathbf{F}(x, y, z) = \langle 2x + 3y, 2y, 2z\rangle$ along the curve parameterized by $\mathbf{r}(t) = \langle t, t^2, t^3\rangle$ for $-1 \le t \le 1$.

28. The following statement is false. If $\mathbf{F}$ is a gradient (i.e. conservative) vector field, then the line integral of $\mathbf{F}$ along every curve is zero. Which single word must be added to make it true?

29. Find a potential function for the vector field $\mathbf{F}(x, y, z) = \left\langle z\sec^2 x, z, y + \tan x \right\rangle$.

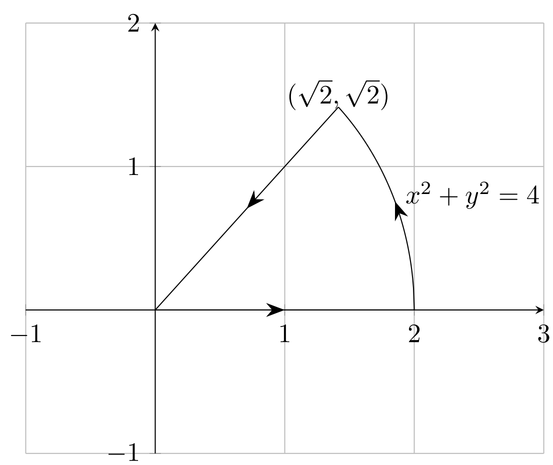
30. Let $f(x,y,z) = x^2yz$, and let $\mathcal{C}$ be a path with parameterization $\mathbf{c}(t) = \left\langle t^2, \sin\frac{\pi t}{4}, e^{t^2-2t} \right\rangle$ for $t \in [0,2]$. Evaluate $\displaystyle\int_{\mathcal{C}} \nabla f \cdot d\mathbf{r}$.

31. For what values of b and c will $\mathbf{F}(x,y,z) = \langle y^2 + 2czx, y(bx+cz), y^2 + cx^2 \rangle$ be a conservative (gradient) vector field?

32. (a) Is $\mathbf{F}(x,y,z) = \langle 2x\cos y, \cos y - x^2 \sin y, z \rangle$ conservative? Justify your answer.

    (b) Calculate $\displaystyle\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$, where $\mathcal{C}$ is the curve $\mathbf{r}(t) = \langle te^t, t\pi, (1+t)^2 \rangle$ for $0 \le t \le 1$.

33. Use Green's Theorem to calculate $\displaystyle\int_{\mathcal{C}} (x-y^3)dx + (x^3-y)dy$ where $\mathcal{C}$ is the closed curve bounding the wedge shaped region pictured on the right.



34. Verify that the vector field $\mathbf{F}(x,y,z) = \langle 2xy + z, x^2 + 1, x + 2z \rangle$ is conservative and calculate the work done by $\mathbf{F}$ in moving an object from $(2,-1,1)$ to $(1,1,0)$.

35. Calculate $\displaystyle\oint_{\mathcal{C}} -y^2\,dx + xy\,dy$ where $\mathcal{C}$ is the counterclockwise oriented simple closed curve consisting of the piece of the parabola $y = 1-x^2$ between $(-1,0)$ and $(1,0)$ together with the piece of the $x-$axis between $(-1,0)$ and $(1,0)$. See graph (right).