

**Changes for the 4th and 5th printings of *A Friendly Intro. To Analysis* Second Edition,
by Witold A. J. Kosmala (updated Oct. 1, 2010)**

- On p. xi, in line 16, change “single-“ to “single”.
- On p. xv, in line 4 of paragraph 2, change “Este” to “Esty”.
- On p. 2 in the middle of the page, the 3rd D should be written as $D = \{x \mid x \text{ and } x + 1, \text{ or } x \text{ and } x - 1 \text{ are prime numbers}\}$.
- On p. 33, Example 1.4.2 should read as follows, “If b is an integer such that b^2 is odd, prove that b is odd.
- On p. 35, Exercise 2 should read as follows, “Prove that if q is an integer such that q^2 is divisible by 3, then so is q .”
- On p. 51, add a period after $x = 1$ on the bottom line.
- On p. 77, in the first line of the paragraph just before Example 2.2.2, replace the second “is” by “can sometimes be”.
- On p. 78, in line 7 of Remark 2.2.5, “ $\lim_{n \rightarrow \infty} \frac{\sin(k/n)}{k/n}$ ” should be changed to “ $\lim_{n \rightarrow \infty} \frac{\sin(k/n)}{k/n} = 1$ ”.
- On p. 78, third line from the bottom, replace “ b_n is between a_n and c_n ” by “ $a_n \leq b_n \leq c_n$ ”.
- On p. 80, in Exercise 17, the second sentence should read as “Find the limit of $\{s_n\}$ by *telescoping the sequence* to just a few terms, the method that could have also been used in Exercises 15 and 16.”
- On p. 83, change the text that comes above Theorem 2.3.6 to the following.

Proof. In determining whether to consider $+\infty$ or $-\infty$, writing out a few terms or simply observing that one of the leading terms has a negative coefficient and the other leading coefficient is positive, suggests that the limit is $-\infty$. Let $M > 0$ be given. We want to find n^* so that for all $n \geq n^*$, we will have $a_n < -M$. But, solving $a_n < -M$ for n is not easy. To avoid this task, we need to bound a_n above by something that tends to $-\infty$. Hence, in this problem we need to make the numerator a larger negative expression, and the denominator a smaller positive expression. Although there are many different choices, let us write

$$-n^3 + 1 < -\frac{1}{2}n^3, \text{ for } n \geq 2, \quad \text{and} \quad n^2 - n - 5 > \frac{1}{2}n^2, \text{ for } n \geq 5.$$

Therefore, picking $n \geq 5$, since the numerator is negative, we write

$$a_n = \frac{-n^3 + 1}{n^2 - n - 5} < \frac{-\frac{1}{2}n^3}{\frac{1}{2}n^2} = -n.$$

But $-n \leq -M$ yields $n \geq M$. Thus, if $n^* \geq \max\{5, M\}$, for all $n \geq n^*$, we have $a_n < -M$. ■

The preceding lengthy proof can be shortened as shown next. Hopefully, this “behind-the-scenes” proof provided insight.

Pick any $M > 0$. Let $n^* \geq \max\{5, M\}$. If $n \geq n^*$, we have

$$a_n = \frac{-n^3 + 1}{n^2 - n - 5} < \frac{-\frac{1}{2}n^3}{\frac{1}{2}n^2} = -n \leq -M.$$

Hence, $\lim_{n \rightarrow \infty} a_n = -\infty$. It should be noted that perhaps showing that $\{-a_n\}$ tends to $+\infty$ and implementing part (d) of Theorem 2.3.3 would be an easier approach. Moreover, since $a_n < -n$, using the comparison test would also prove the divergence to $-\infty$.

See Exercises 3 and 9 for more information concerning rational expressions. There are other ways to determine divergence to infinity. The next result relates ideas from previous sections to the divergence to infinity.

- On p. 87, in Exercise 14, in the limit, change “ B ” to “ β ”.
- On p. 110, in Exercise 5 should read as “Prove that every unbounded above sequence contains a monotone subsequence that diverges to plus infinity.”

- On p. 118, in Definition 3.1.2, between words “then” and “the”, add the following: “ f has a *horizontal asymptote at $+\infty$* and”.
- On p. 118, Example 3.1.3 should read as follows: “Assume $D = \mathbb{R} \setminus Q$ and $f : D \rightarrow \mathbb{R}$ be a function defined by $f(x) = \frac{x}{x+2}$. Verify that $\lim_{x \rightarrow \infty} \frac{x}{x+2} = 1$.” In the proof of Example 3.1.3 change “ Q^+ ” to “ $\mathbb{R} \setminus Q$ ” in two places.
- On p. 121, two lines above Definition 3.1.10, change “integers” to “natural numbers”.
- On p. 123, 3 lines above Exercises 3.1 begin, change the word “diverge” to “tend”.
- On p. 126, in Figure 3.2.1, fill in the point $(a, f(a))$ and make the vertical line above $a + \delta$ dotted.
- On p. 138, in the last sentence of the section, remove the word “that”.
- On p. 141, in Exercise 19, change “ $\{a_n\}$ converges” to “ $\{a_n\}_{n=2}^\infty$ converges and prove your result”.
- On p. 142, in Exercise 21, add to the sentence “, where $x \in \mathbb{R}$ ”.
- On p. 158, in part (b) of Definition 4.2.3, change “ $a \notin D$ ” to “ $a \in D$ ”.
- On p. 174, in part (c) of Exercise 5, delete what is in the parentheses.
- On p. 187 in Example 5.1.8, change “ $x = a$ ” to “ $x = 0$ ”.
- On p. 224 in last line change \sqrt{x} to $x^{\frac{1}{x}}$.
- On p. 225 in Exercise 1(n) change \sqrt{x} to $x^{\frac{1}{x}}$.
- On p. 226 in Exercise 7 change \sqrt{x} to $x^{\frac{1}{x}}$.
- On p. 227 in Exercise 12 change \sqrt{a} and \sqrt{b} to $a^{\frac{1}{x}}$ and $b^{\frac{1}{x}}$.
- On p. 271, in Exercise 9, change the last word “finite” to “convergent”.
- On p. 344, in the 3rd line in Proof of part (b), change “converges to 1 sufficiently fast. Suppose that” to “converges to 1, say,”.
- On p. 350, in Exercise 2 add at the end “(Do not use Theorem 8.3.4.)”
- On p. 350, in Exercise 4 add at the end “for the increasing case.”
- On p. 350, in Exercise 5(c), change “ $f_n(x) \leq f_{n+1}(x)$ ” to “ $f_n(x) \leq f_{n+1}(x)$ (or $f_n(x) \geq f_{n+1}(x)$)”.
- On p. 362, in part (b) of Theorem 8.5.8, change $<$ to \leq .
- On p. 363, in parts (a) and (b) of Theorem 8.5.12, change the end of the lines from “for any $\varepsilon_1 > 0$ ” to “for any small enough $\varepsilon_1 > 0$ ” and from “for any $\varepsilon_2 > 0$ ” to “for any small enough $\varepsilon_2 > 0$ ”.
- On p. 410, two lines above Theorem 9.6.10, change the sentence “Clearly, \vec{v} has no tangent line when $t = 0$.” to “Clearly, \vec{v} has no tangent line when $t = 0$, and a particle traveling along this curve would be at a stand-still when $t = 0$.”
- On p. 458, change the last sentence before Definition 10.4.3 to “When f is differentiable, then the total derivative \bar{m} is given by the gradient, which is defined next.”
- On p. 460, in Exercise 1, part (c), change “have vertical” to “have a vertical”.
- On p. 517 in Exercise 8 change “asteroid” to “astroid”.
- On p. 538, change part (h) of Exercise 11 to: “Since the sequence is decreasing and bounded below by 0, it is convergent. However, taking limits of the recursion formula will not give the value of the limit. See Exercise 7(f) ... “.
- On p. 558 in Section 11.5 remove answers to Exercises 11 and 12.
- On p. 574, change “Witch of Agnesi, 340” to “Witch of Agnesi, 340, 401”.