Changes for the 6th printing of *A Friendly Intro. To Analysis* Second Edition, by Witold A. J. Kosmala (updated Oct. 1, 2010)

My apologies that the book is with a soft cover. This was publisher's choice. I know that the binding breaks when one fully opens the book for study purposes. If that is a problem for you because pages start falling out, you might wish to go to a local print shop and have them rebind the book with a coil.

On p. 2 in the middle of the page, the 3rd D should be written as $D = \{x \mid x \text{ and } x+1, \text{ or } x \text{ and } x-1 \text{ are prime numbers}\}$.
On p. 14 publisher chose to put page 32 instead. Below is a correct page 14.
On p. 187 in Example 5.1.8, change " $x = a$ " to " $x = 0$ ".
On p. 224 in last line change $\sqrt[x]{x}$ to $x^{\frac{1}{x}}$.
On p. 225 in Exercise 1(n) change $\sqrt[x]{x}$ to $x^{\frac{1}{x}}$.
On p. 226 in Exercise 7 change $\sqrt[x]{x}$ to $x^{\frac{1}{x}}$.
On p. 227 in Exercise 12 change $\sqrt[x]{a}$ and $\sqrt[x]{b}$ to $a^{\frac{1}{x}}$ and $b^{\frac{1}{x}}$.
On p. 344, in the 3rd line in Proof of part (b), change "converges to 1 sufficiently fast. Suppose that" to "converges to 1, say,".
On p. 517 in Exercise 8 change "asteroid" to "astroid".
On p. 558 in Section 11.5 remove answers to Exercises 11 and 12.

supremum, of f and is denoted by $\sup f = \sup_{x \in A} f(x)$. The largest of all lower bounds, if a lower bound exists, is called the *greatest lower bound*, or *infimum*, of f and is denoted by $\inf f = \inf_{x \in A} f(x)$.

A function f is bounded if and only if $|f|$ is bounded above. Observe that since for any function $f : A \to B$, $f(A)$ is a set of real numbers, Definition 1.2.14 applies to any set $S \subset \Re$. A nonempty set $S \subset \Re$ is *bounded above* if and only if there exists a real number M_1 such that $x \leq M_1$ for all $x \in S$. A nonempty set S is *bounded below* if and only if there exists $M_2 \in \Re$ such that $x \geq M_2$ for all $x \in S$. A nonempty set S is *bounded* if and only if it is bounded above and bounded below. The supremum and infimum of a nonempty set are defined the same way as the supremum and infimum of a function. See Theorem 1.7.7 for more about the supremum of a set.

Definition 1.2.15. Consider a function $f : A \to B$.

(a) A point $x_0 \in A$ is a *root* of f if and only if $f(x_0) = 0$. That is a value where the graph of f intersects the horizontal axis. We say that f *vanishes* at $x_0 \in A$ if and only if x_0 is a root of f. In addition, f is *identically zero*, denoted by $f(x) \equiv 0$, if and only if $f(x) = 0$ for all $x \in A$. If $f(x) \equiv 0$, then f is called a *zero function*.

(b) The function f has an *absolute (global) maximum*, (or simply *maximum*), at a value $x_1 \in A$ if and only if $f(x) \leq f(x_1)$ for all $x \in A$. The value $f(x_1)$ is the *absolute (global) maximum*, (or simply *maximum*), of f and is denoted by $\max_{x \in A} f(x)$ or by $\max f$.

(c) The function f has an *absolute (global) minimum* (or simply *minimum*) at a value $x_2 \in A$ if and only if $f(x) \geq f(x_2)$ for all $x \in A$. The value $f(x_2)$ is the *absolute (global) minimum* (or simply *minimum*) of f and is denoted by $\min_{x \in A} f(x)$ or by $\min f$.

(d) If $B = A$ and $f(x) = x$ for all $x \in A$, then f is called an *identity function*.

A function f has an *extremum* at $x = x_1$ if and only if it has a maximum or a minimum at $x = x_1$. If $\max f$ exists and equals a value M, then $\sup f = M$ (see Exercise 5).

Example 1.2.16.

(a) If a function $f : (0, \infty) \to \Re$ is defined by $f(x) = \dfrac{x-1}{x}$, then f is bounded above, not bounded below, $\sup f = 1$, $\max f$ does not exist, and $x = 1$ is a root of f (see Figure 1.2.7).

(b) Consider the function $g : \Re \to \Re$ defined by $g(x) = \dfrac{1}{x^2+1}$. Graph g and try to convince yourself that g is bounded, $\sup g = \max g = 1$, $\inf g = 0$, and $\min g$ does not exist. In addition, g has no roots. □

Definition 1.2.17.

(a) A function $f : \Re \to \Re$ is called a *polynomial* if and only if it can be written as

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$$

where n is some fixed nonnegative integer called the *degree (order)* of the polynomial, and a_i, for $i = 0, 1, 2, \ldots, n$, is a real number called a *coefficient* with $a_n \neq 0$. The