Advanced Calculus, exam on Kirkwood, Chapter 6 (Integration) Name__________________
Spring 2014 (initial each sheet you use)

**Read these instructions!** Demonstrate that you know how proofs and disproofs are written. If you did it on the homework, or it was done in the text, do it again here. Do not use more-sophisticated results to prove less-sophisticated results.

1. (12 pts) Examples (just give the example, not the proof):

a) Let *f* be non-negative and integrable on [*a*, *b*]. Give an example such that, for all partitions P, U(P, *f*) > 0, but $\int_a^b f(x)dx = 0$.

b) Usually the conclusion of Version 2 of the Fundamental Theorem of Calculus holds, but not always. Give an example such that *f* is Riemann integrable on [*a*, *b*] and

$F(x) = \int_a^x f(t)dt$ but $F'(x) \neq f(x)$.

2. (12 pts) [Short answer, no work required]

a) Which differentiation rule corresponds to integration by parts?

b) Which differentiation rule corresponds to integration by substitution (= integration by change of variables)?

c) Find $\frac{d}{dx}\int_0^{x^2} \cos(t^3)dt$.

3. (12 pts) On [0, 1], let $f(x) = 1$ if $x = 1/n$ for $n = 1, 2, 3, \ldots$ and $f(x) = 0$ otherwise. It has in infinite number of discontinuities. Nevertheless, *f* is Riemann integrable. Prove it.

4. (14 pts) Prove: If *f* is integrable on [*a*, *b*] then $\int_a^x f(t)dt$ is continuous at *x* for $a < x < b$.

5. (15 points) Prove **one** of these two:

Option 4A) Continuous functions on [*a*, *b*] are integrable there.

Option 4B) Increasing functions on [*a*, *b*] are integrable there.

6. (25 points) Do **one** of Option A or Option B. For the one you choose do two things:

a) Restate it properly with its hypotheses and then b) prove it.

Option A) $\int f = F(b) - F(a)$

Option B) $\frac{d}{dx}\int f = f(x)$

7. (10 pts) Asymptotically, about how large is $\sum_{k=1}^{n}\frac{1}{n^2+k^2}$ ? [Find a simpler function asymptotic to it for large *n*.]