Introduction to Analysis, II. Spring 2014 Name ______________________
Series (initial each sheet)

1.(16 pts) State and prove the formula for the sum of a general geometric series.

2.(16 pts) a) Draw a picture illuminating the integral test and state the key inequalities for a_n.

b) Use them to find values b and c such that $b < \sum_{n=100}^{\infty} \frac{1}{n^3} < c$.

3. (6 pts) [Short answer. Give the requested idea, not the full proof] Suppose we already have sequence theorems, and we want to prove the Comparison Test for Series: If $0 \le a_n \le b_n$ for all n and $\sum b_n$ converges, then $\sum a_n$ converges. What is the key sequence result that allows the proof?

4. (10 pts) [Give the requested ideas, not the full proof] Suppose $a_n > 0$ for all n and $\lim \frac{a_{n+1}}{a_n} = r < 1$. Then $\sum a_n$ converges. There are two key ideas in the proof. What are they?

5. (6 pts) Find $\{a_k\}$ such that $\sum_{k=1}^{n} a_k = \frac{(-1)^n}{n}$. [After you answer, do not bother to simplify.]

6. (8 pts) Complete the sentence. [It will have two parts looking somewhat like limit definitions.]
"lim sup $a_n = L$ iff for each $\varepsilon > 0$ ...".

7. (12 pts) a) Give the definition of "telescoping series." What does it mean for $\sum a_n$ to be "telescoping"? Give a formal definition. [Ideas that show you know what it means but are not formalized will be worth partial credit.] b) Use your terms to express the partial sums, $\sum_{k=1}^{n} a_k$, when the series is telescoping.

*** The next two conjectures are **false. Construct specific counterexamples**. (10 pts each)

Conjecture 8: If $\sum a_k$ converges and $\sum b_k$ converges, then $\sum a_k b_k$ converges.

Conjecture 9: Suppose $a_n > 0$ for all n. If for each n^* there exists $n > n^*$ such that $a_{n+1}/a_n > 2$, then $\sum a_k$ diverges.

10. (6 pts) True or false? [No reason required. This is the conjecture we studied in the homework.]

T F If a_n and b_n are never zero and $\frac{a_n}{b_n} \to 1$, then $\sum a_k$ and $\sum b_k$ converge or diverge together.