The Limit Comparison Test

Another student comes to you and says "I can prove the limit comparison test if both series have only positive terms. You know, if $\lim \frac{a_n}{b_n} = L$, where $0 < L < \infty$, then the two series $\sum a_n$ and $\sum b_n$ converge or diverge together. Is there a limit comparison text if we don't have the hypothesis about positive terms?"

Fortunately, before you have to answer, you are rescued by the need to go to another class. Then you decide to make a good impression by responding with a thorough answer the next day.

Give a complete answer to that question. Justify all your assertions.
You must
a) State interesting conjectures, and
b) resolve them.