

### Types of Errors and Power – Review of Activity 12:

- If the p-value is less than the significance level ( $\alpha$ )
  - The decision is to reject the null hypothesis
- If the p-value is greater than the significance level ( $\alpha$ )
  - The decision is to fail to reject the null hypothesis

Decisions  
– p-value  $< \alpha$   
– reject  $H_0$   
– p-value  $> \alpha$   
– fail to reject  $H_0$

#### Type I Error

- Reject the null hypothesis, when the null is actually true
- Can only be made when the *p-value is smaller than the significance level*.
  - Possible when we reject the null hypothesis

#### Type II Error

- Fail to reject the null hypothesis, when the null is actually false
- Can only be made when the *p-value is larger than the significance level*.
  - Possible when we fail to reject the null hypothesis

$\alpha$  – alpha  
– significance level

#### Power

- Probability of rejecting the null, when the null is false.

#### Writing errors in context.

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- Type 1: Concluding there is evidence in support of the alternative hypothesis, when in fact the null hypothesis is true
  - Type 2: Concluding there is little to no evidence in support of the alternative hypothesis, when in fact the null hypothesis is false.
  - Power: Probability of concluding there is evidence in support of the alternative hypothesis, when in fact the null hypothesis is false.

#### Example Module 6 and 7 Lab:

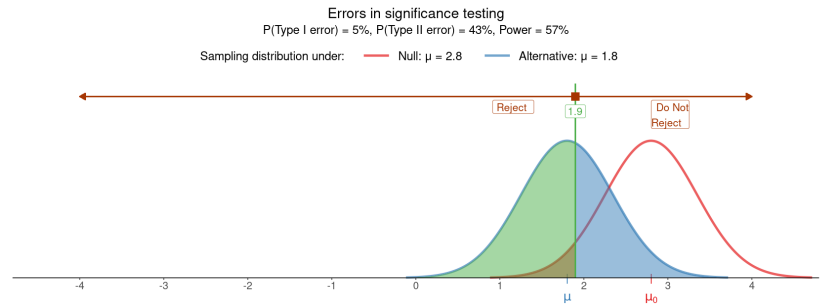
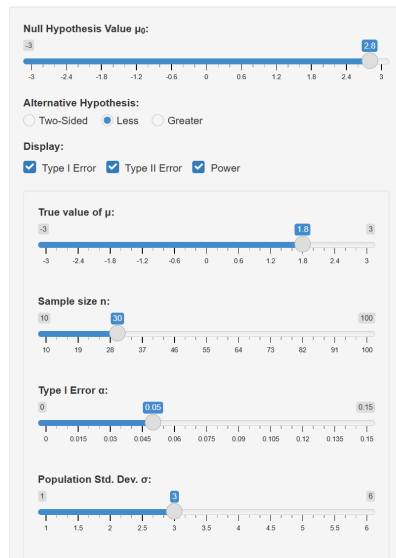
- Type 1: Concluding there is evidence that the true mean arsenic level in New Hampshire residents with private wells is greater than 0.150 ppm, when in fact the true mean arsenic level is 0.150 ppm.
- Type 2: Concluding there is little to no evidence that the true mean arsenic level in New Hampshire residents with private wells is greater than 0.150 ppm, when in fact it is greater than 0.150 ppm.
- *Note: Since the results gave a very small p-value and the 90% confidence interval did not contain the null value of 0.150, we would reject the null hypothesis and could only have the possibility of a Type I error.*
- Power: Probability of concluding there is evidence that the true mean arsenic level in New Hampshire residents with private wells is greater than 0.150 ppm, when in fact it is greater than 0.150 ppm.

## Review of Errors and Power

### Activity 12:

$\mu$  represents the true mean cost of textbook for college students in dollars

$$H_0: \mu = \$280, H_A: \mu < \$280$$

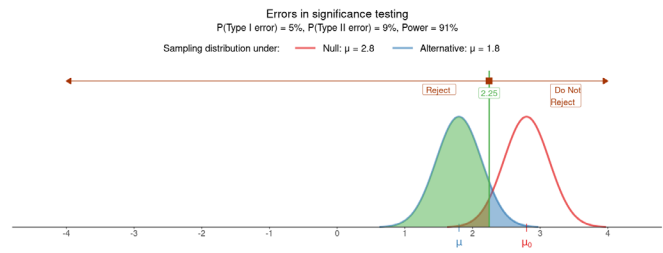
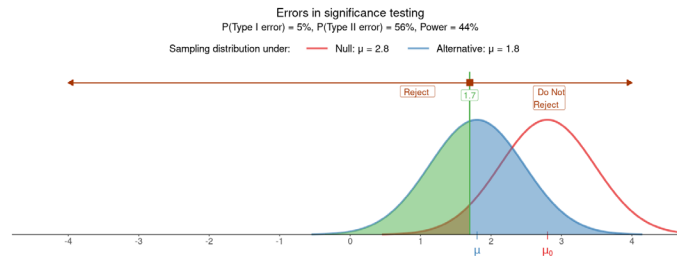
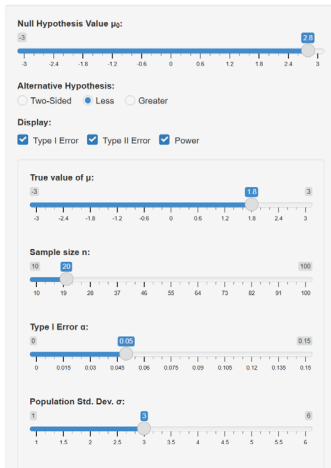


- In this illustration, we are assuming that the null value is 2.8 (\$280) and testing that the believed true mean is 1.8 (\$180). The red distribution represents the null hypothesis is true and the blue distribution that the null hypothesis is false.
- The shaded in red area to the left of the value of 1.9 under the red distribution represents the Type I Error – rejecting the null, when the null is true. The probability of this error is 5% (the significance level).
- The shaded in blue area to the right of the value of 1.9 under the blue distribution represents the Type II Error – failing to reject the null, when the null is false.
- The shaded in green area to the left of the value of 1.9 under the blue distribution represents the Power – the probability of rejecting the null, when the null is false.
- Since the Power and the Type II Error are both conditional on the null being false ->  $\text{Power} + P(\text{Type II Error}) = 1$

## Review of Errors and Power

**How do we increase the probability of rejecting  $H_0$  (finding strong evidence for  $H_a$ ) when  $H_a$  is true? How do we increase power?**

■ **↑ sample size**

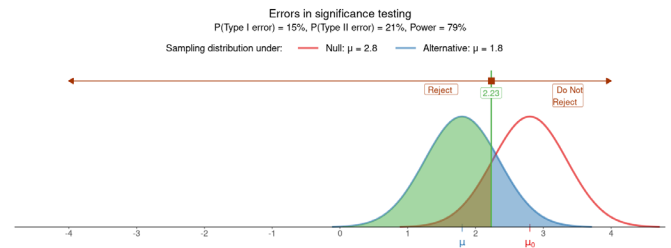
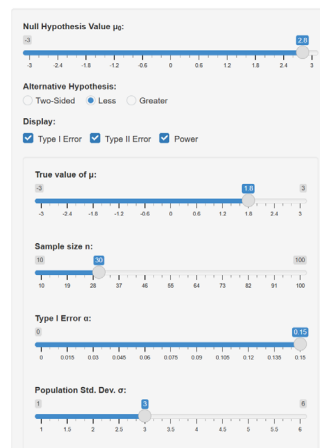
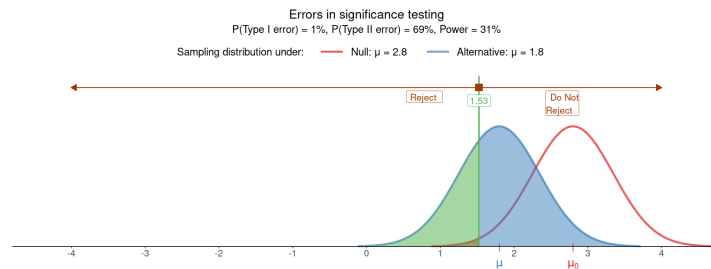
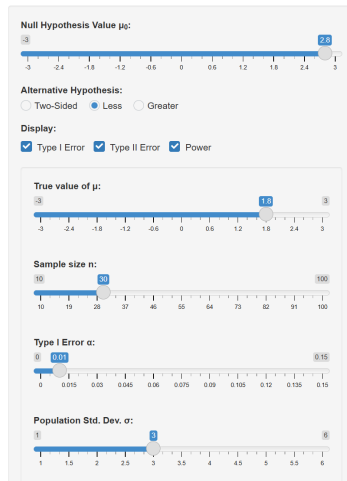


- **Note that increasing sample size results in less sample to sample variability so the distributions are not as spread out when we increase the sample size.**
- **Increasing the sample size -> increases the power of the test.**

## Review of Errors and Power

**How do we increase the probability of rejecting  $H_0$  (finding strong evidence for  $H_a$ ) when  $H_a$  is true? How do we increase power?**

■ **↑ significance level** *P(Type I)*

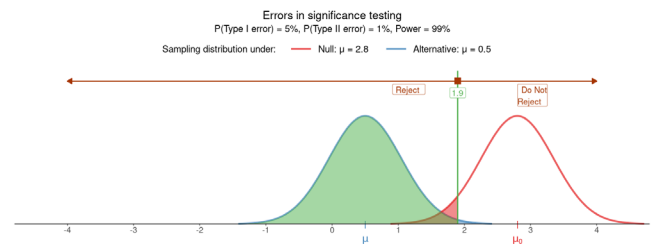
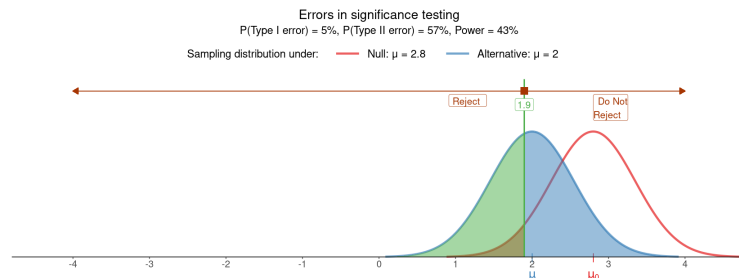
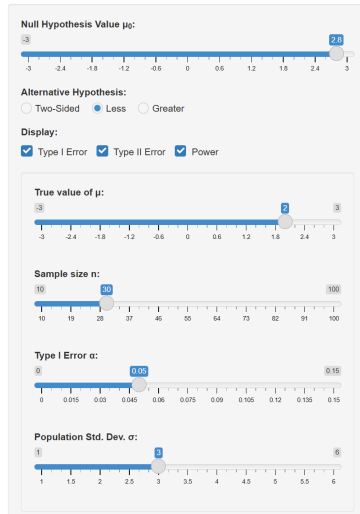


- Note that increasing the Type I Error (the significance level) moves the value of  $\bar{x}$  at which point we will reject the null closer to the null value. We can reject the null for larger values of  $\bar{x}$  because we have increased the significance level (0.01 to 0.15).
- Increasing the significance level  $\rightarrow$  increases the power of the test.

## Review of Errors and Power

**How do we increase the probability of rejecting  $H_0$  (finding strong evidence for  $H_a$ ) when  $H_a$  is true? How do we **increase power**?**

■ **↑** difference between  $H_0$  and the believed true probability of successes (**effect size**)



- Note that when we increase the effect size (difference between the null value and the believed true value) the red and blue distributions move farther apart increasing the power of the test.
- Increasing the effect size  $\rightarrow$  increases the Power of the test.