Review of Module 7

Parameter of Interest: What information do we want to know about the population?:

- The parameter of interest is used in the hypotheses statements, in conclusions, and in many interpretations!

Include:

- Reference of the population (true, long-run, population, all)
    - Clearly refer to the population
- Summary measure (mean)
    - What numerical value are we calculating?
    - This is dependent on the type of variable(s) in our study
- Context
    - Observational units/cases – what or whom are we collecting data on
    - Variable of interest

Example Activity 11:

- ***μ represents the true mean body temperature (in °F) of Stat 216 undergraduates***

Hypothesis Test (test of significance/inference): test to show evidence based on the sample statistic against the null hypothesis

Hypotheses:

- Null Hypothesis: This is the known claim that we are trying to disprove; may be based on random chance
    - H_0: $\mu = 0$
- Alternative: this is the claim we are testing that is based on the research question
    - H_A: $\mu \begin{Bmatrix} > \\ \neq \\ < \end{Bmatrix} 0$
        - The direction of the alternative (the sign) is determined by the research question
- **Example Activity 11:** Is there evidence that MSU students get less than the recommended 7 hours of sleep per night, on average?
    - *H_0: **The true mean body temperature (in °F) of Stat 216 undergraduates is 98.6 °F.***
        - *Note: we are assuming that the the average body temperature of Stat 216 undergraduate students is the same as the typical temperature of 98.6.*
        - $H_o: \mu = \mathbf{98.6°F}$
    - *H_A: **The true mean body temperature (in °F) of Stat 216 undergraduates differs from 98.6 °F.***
        - *The direction of the alternative is less note equal to because the research question asks for evidence that the mean body temperature differs from 98.6*
        - $H_A:\ \mu \neq\ \mathbf{98.6°F}$

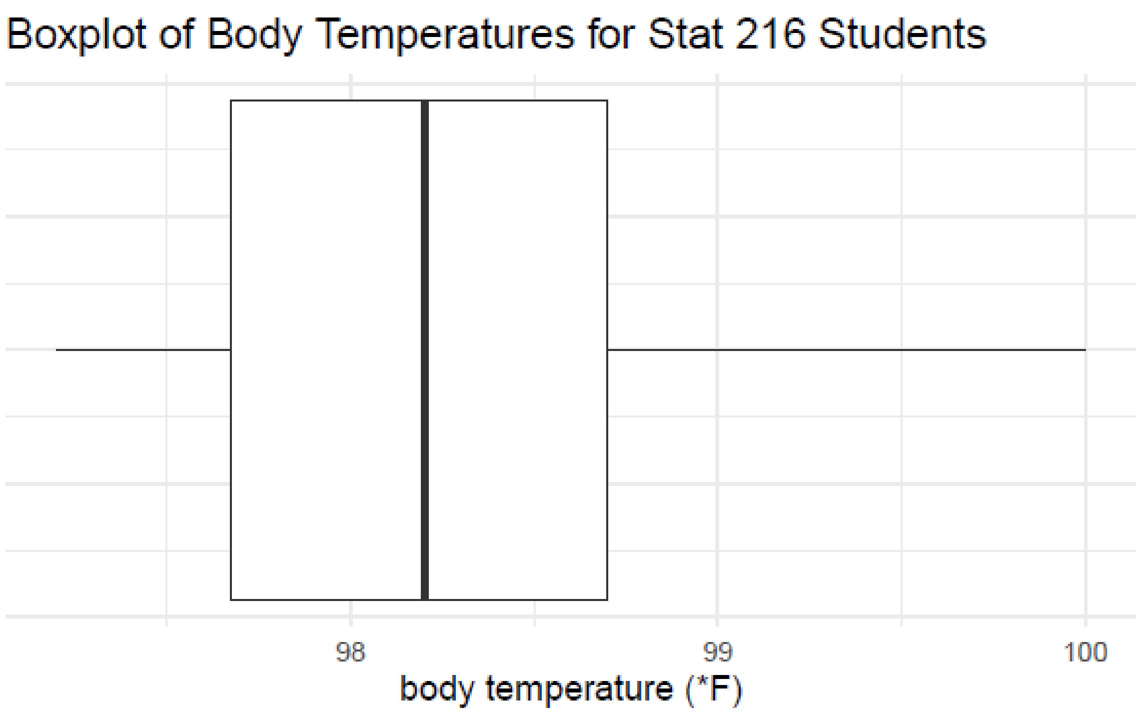
Conditions for the sampling distribution of $\bar{x}$ to follow an approximate normal distribution:

- Independence: the sample's observations are independent, e.g., are from a simple random sample. (*Remember*: This also must be true to use simulation methods!)
- Normality Condition: either the sample observations come from a normally distributed population — approximately symmetric
  or we have a large enough sample size. To check this condition, use the following rules of thumb:
  - $n < 30$: The distribution of the sample must be approximately normal with no outliers.
  - $30 \leq n < 100$: We can relax the condition a little; the distribution of the sample must have no extreme outliers or skewness.
  - $n \geq 100$: Can assume the sampling distribution of $\bar{x}$ is nearly normal, even if the underlying distribution of individual observations is not.

Is the sample size large enough?

- t-distribution: a theoretical distribution that is bell-shaped with mean zero. Its degrees of freedom determine the variability of the distribution. For very large degrees of freedom, the t-distribution is close to a standard normal distribution. For a single quantitative variable, the degrees of freedom are calculated by subtracting one from the sample size: $n - 1$. A t-distribution with $n - 1$ degrees of freedom is denoted by: t_{n-1}.

Example Activity 11:




```
#>     min      Q1 median   Q3 max     mean        sd  n missing
#> 1 97.2 97.675   98.2 98.7 100 98.28462 0.6823789 52       0
```

- Since the sample size is between 30 and 100 (n = 52) and the distribution of body temperatures has no extreme outliers, the theory-based p-value will be approximately the same as the simulation p-value

Review of Module 7

mean

Calculation of the Standardized Statistic:

- For a theory-based hypothesis test first we will calculate the standardized statistic. The standardized statistic is compared to the t-distribution with n-1 degrees of freedom (df) to find the p-value.

$$T = \frac{\bar{x} - \mu_0}{SE(\bar{x})}$$

$$SE(\bar{x}) = \frac{s}{\sqrt{n}}$$

Example Activity 11: $\bar{x} = 98.2846, s = 0.6824, n = 52$

$$SE(\bar{x}) = \frac{0.6824}{\sqrt{52}} = 0.0946$$

Interpretation of the standard error of the sample mean: The standard error of the sample mean measures the on average distance each sample mean is from the true mean.

Include:

- Summary measure (in context)
- On average distance from parameter or null value

Example Activity 11: statistic

- Each sample mean body temperature for MSU undergraduate students from a sample of 52 MSU students is 0.0946°F, on average, from the true mean body temperature.

Calculation of the standardized sample mean for Activity 11:

$$T = \frac{98.2846 - 98.6}{0.0946} = -3.334$$

Interpretation of the standardized statistic: *measures the number of standard errors the sample statistic is from the null value*

Include:

- Statistic (summary measure and value in context)
- Units of measure are standard errors *(value of T)*
- Above/below *the null value (give the value)*

Example Activity 11:

- *The sample mean body temperature for MSU undergraduates of 98.846 °F is 3.334 standard errors below the null value of 98.6°F.*

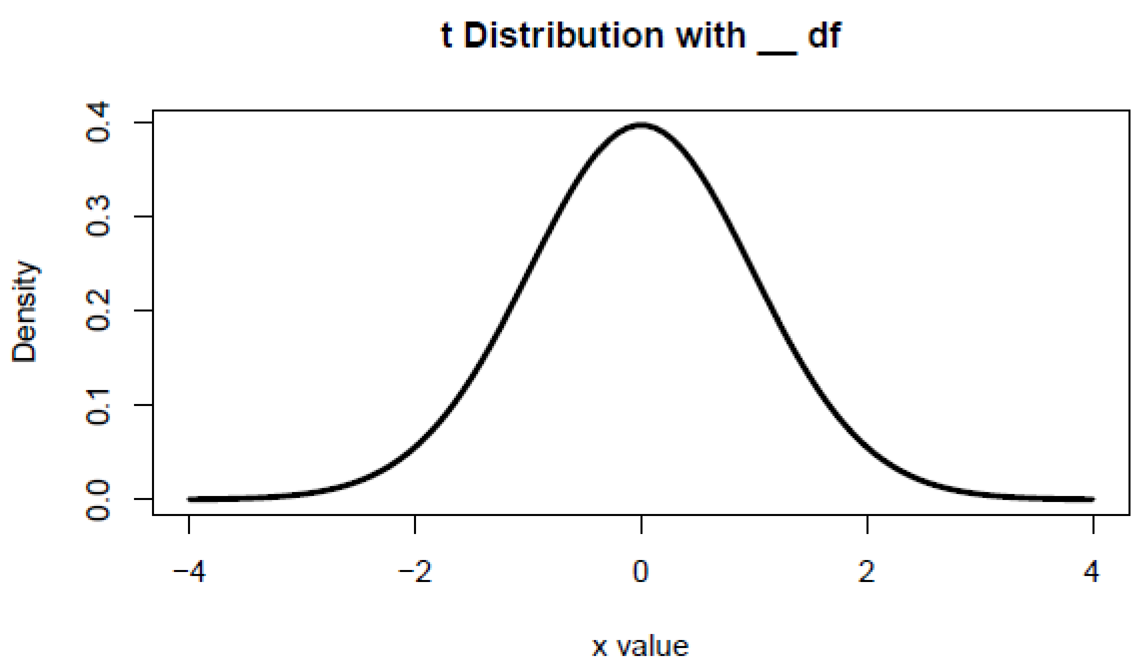
To find the p-value:

Example Activity 11:

The pt function in Rstudio is used to find the p-value:

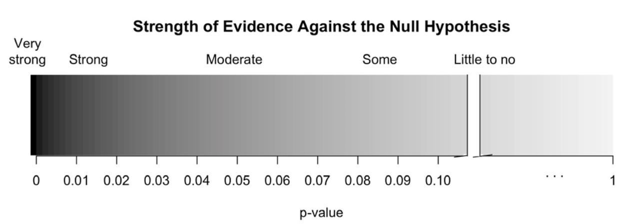
```
2*pt(-3.334, df=51, lower.tail=TRUE)
    [1] 0.001583532
```

T-distribution with 51 df:



Strength of Evidence: How much evidence does the p-value provide **against** the null?

- Use the guidelines for the strength of evidence



- The smaller the p-value the MORE evidence there is against the null hypothesis

Review of Module 7

There are FOUR things we ask about the p-value (we will learn the 4th in Module 7)

Evaluation of a p-value:

- How much evidence does the p-value provide AGAINST the null hypothesis?
- **Example Activity 11:** The theory-based p-value for this study was found to be 0.0016.
    - ***There is very strong evidence against the null hypothesis that the true mean body temperature for Stat 216 undergraduates is 98.6 °F.***

Interpretation of a p-value:

- What the p-value measures: the probability of observing the sample statistic or more extreme if the null hypothesis is true (Don't forget the context!)
- Include in the interpretation:
    - Statement about probability (in x% of simulated samples, in x out of 1000 simulated samples, with a probability of x%)
    - Statistic in context (give the value and in words what the statistic represents)
    - more extreme (direction of the alternative)
    - If the null hypothesis is true in context (give the null value and in words what the null represents)
        - Note: context only needs to be included in either the statistic OR the null

- **Example Activity 11:** The theory-based p-value for this study was found to be 0.0016.
    - ***We would observe a sample mean of 98.6246 °F or more extreme in both tails with a probability of 0.0016 if we assume the true mean body temperature for Stat 216 undergraduates is 98.6 °F.***

    OR

    - ***If the true mean body temperature for Stat 216 undergraduates is 98.6 °F we would observe a sample mean of 98.6246 °F or more extreme in both tails with a probability of 0.0016.***

    OR

    - ***There is a 0.16% chance, we would observe a sample mean 98.6246 °F or more extreme in both tails, if we assume the true mean body temperature for Stat 216 undergraduates is 98.6 °F.***

Conclusion: Answers the research question. Write a conclusion as the amount of evidence in support of the alternative.

- **Example Activity 11**: The theory-based p-value for this study was found to be 0.0016.
    - ***There is very strong evidence** that the **true mean body temperature for Stat 216 undergraduates differs from 98.6 °F.***

Calculation of the Confidence Interval Using Theory-based Methods:

- To calculate the confidence interval we add and subtract the margin of error to the point estimate.

$$\boldsymbol{point\ estimate \pm margin\ of\ error}$$

- The margin of error is the t* multiplier times the standard error

$$ME = t^* \times SE(\bar{x})$$

statistic

$$\bar{x} \pm \boxed{t^* \times SE(\bar{x})}$$

$$\boldsymbol{SE(\bar{x}) = \frac{s}{\sqrt{n}}}$$

- The t* multiplier – determines the width of the confidence interval and is NOT the same value as T (the standardized statistic).
  - *Dependent on the level of confidence and the sample size*
  - *We use the t-distribution with n – 1 degrees of freedom*

Example Activity 11: $\bar{x} = \mathbf{98.28462}, \boldsymbol{s} = \mathbf{0.6823789}, \boldsymbol{n} = \mathbf{52}$

- To find a 95% confidence interval we will use a multiplier of t* = 2.008

  qt(0.975, df = 51, lower.tail = TRUE) = 2.008

$$\boldsymbol{SE(\bar{x}) = \frac{0.6824}{\sqrt{52}} = 0.0946}$$

$$ME = 2.008 \times 0.0946$$

$$98.285 \pm 0.190$$

$$(98.095,\ 98.475)$$

Interpretation of a confidence interval:

- Include in the interpretation:
  - How confident you are (90%, 95%, 99%)
  - Parameter of interest *in context*
    - *Population word (true, long-run, population)*
    - *Summary measure (difference in proportion)*
    - *Observational units*
    - *Variable of interest*
  - Calculated Interval

Example Activity 11: ***We are 95% confident, the true mean body temperature for Stat 216 undergraduates is between 98.095 and 98.475 °F.***