Name ________________________ Score __________________

MATH 581 - Fall 2004 - Homework 2

Carefully Read and Follow Directions Clearly label your work and attach it to this sheet. No credit will be given for unsubstantiated answers.

1. In your textbook on page 19, show that equation (2.54) is true. That is, show that there exists a constant $C(\nu)$ depending only on the value of ν such that

$$|\lambda(k) - e^{-k^2\Delta t}| \leq C(\nu)\ k^4(\Delta t)^2,$$

   for all k and $\Delta t > 0$.

2. In class, we have discussed the idea of stability for the explicit numerical scheme. Reproduce the results of stable and unstable calculations in Figure 2.2. For the initial condition given in (2.24), note that the fourier series expansion of the solution of the model problem is given by

$$u(x,t) = \frac{8}{\pi^2}\sum_{m=1}^{\infty}\frac{1}{m^2}\sin(m\pi/2)\sin(m\pi x)e^{-m^2\pi^2 t},$$

   in order to plot the error at a given time step, one can choose to use the first 10 or 20 terms of this infinite series as a good approximation of true solution.

3. Consider the pde given by

$$u_t = (a(x)u_x)_x, \quad 0 < x < 1$$

$$u(0,t) = u(1,t) = 0, \quad t > 0$$
$$u(x,0) = u^0(x), \quad 0 \leq x \leq 1$$

   where $a(x) > 0$ for $0 \leq x \leq 1$. Use the forward time difference and the first order central difference operator in space (applying it twice) in order to derive an explicit numerical scheme to approximate the solution to the given equation.