Name ________________________ Score ____________________

MATH 581 - Fall 2004 - Homework 4

Carefully Read and Follow Directions Clearly label your work and attach it to this sheet. No credit will be given for unsubstantiated answers.

1. Assume $\epsilon > 0$ for this problem.

$$\begin{aligned} u_t &= \epsilon u_{xx}, \quad x \in (0,2), t > 0 \\ u(x,0) &= \cos\frac{\pi x}{2}, \quad x \in [0,2] \\ u_x(0,t) &= u_x(2,t) = 0, \quad t \geq 0. \end{aligned}$$

Note that the true solution to this pde is given by

$$u(x,t) = \cos(\frac{\pi x}{2}) e^{-(\frac{\pi}{2})^2 \epsilon t}.$$

In class, we have discussed a couple of approaches to the Neumann boundary conditions. One was the first-order difference approximation to the derivative u_x. The second entailed the use of the "ghost point".

(a) For each of these approaches, develop a Crank-Nicolson scheme for this problem. Describe the scheme, and in particular, give the relations which describe the unknowns at the boundaries. (ie. $u_0^{n+1} =?$ and $u_J^{n+1} =?$) Be sure to also describe the size and form of the system matrix and right hand side vector for each case.

(b) Implement the scheme numerically for each of the approaches. Compare the performance of each algorithm. Give error plots, and determine which method seems to work better.