Name ________________________ Score ____________________

Spring 2005

MATH 582 Homework 4

Carefully Read and Follow Directions Clearly label your work and attach it to this sheet. No credit will be given for unsubstantiated answers.

1. Show that

$$\sup_{0<\xi<1} \int_0^1 |y - \xi| dy = \frac{1}{2}$$

(This result was used in the proof of Theorem 0.4.5 in the notes from the Brenner and Scott handout.)

2. In Homework 3, problem 1c, you derived the general form of the stiffness matrix for the boundary value problem

$$\begin{aligned} -u''(x) &= f(x), \quad x \in (0,1), \\ u(0) = 0, & \qquad u(1) = 0, \end{aligned}$$

   (a) Create a Matlab subroutine that creates the stiffness matrix for a given set of grid points. Assume a non-uniform step size. Hence, one will be given a set of grid points, and the code should compute a corresponding vector of h_i values. (test your matrix by using first a uniform grid size and second by using only a small number of grid points in a non-uniform grid).

   (b) Next generate a test problem. Choose the true solution $u(x) = \sin(2\pi x)$, and show that u satisfies the boundary conditions of the bvp. Give the form of the right-hand side $f(x)$ which forces u to be a solution to the bvp.

   (c) Returning to the weak form that you gave in Homework 3, 1a, give the form of the right-hand side vector that must be constructed in order to numerically approximate the solution to the bvp for the test problem. When the "hat" functions (see Homework 3) are used as the basis functions for the test problem, the integral form can be simplified to an algebraic expression. Give that explicit expression.

   (d) Next create a Matlab subroutine that generates the right-hand side vector that you just computed above.

   (e) Finally, use Matlab's "backslash" command or your favorite linear solver to approximate u using the "hat" functions as the basis. Compute the error (just use the discrete 2-norm), and construct a table to show that the absolute error $\to 0$ as $h \to 0$. Also generate a plot of the true soln and the computed approximation.