

## Third Practice Test, MATH 224, Fall 2007

1. Calculate  $\int_0^1 \int_{\sqrt{y}}^1 \frac{ye^{x^2}}{x^3} dx dy$  by reversing the order of integration.
2. Consider a spherical shell  $E$  between the spheres  $x^2 + y^2 + z^2 = 1$  and  $x^2 + y^2 + z^2 = 4$  with mass density equal to the distance to  $(0, 0, 0)$ . Find the total mass.
3. Calculate  $\int_C y^3 ds$ , where  $C$  is the part of the graph  $y = 2x^3$  from  $(0, 0)$  to  $(1, 2)$ .
4. Which of the following vector fields are conservative? Find a potential for one of them and use it to calculate  $\int_C \mathbf{F} \cdot d\mathbf{r}$  where  $C$  is the arc of the unit circle from  $(1, 0)$  to  $(0, 1)$  in counterclockwise direction.

$$\mathbf{F}_1(x, y) = \langle x^2, x^2 \rangle$$

$$\mathbf{F}_2(x, y) = \langle 2xy, x^2 \rangle$$

$$\mathbf{F}_3(x, y) = \langle e^y, e^x \rangle$$

$$\mathbf{F}_4(x, y) = \langle e^x, e^y \rangle$$

5. Use Green's Theorem to evaluate  $\int_C \sqrt{1+x^2} dx + x(1+\sin y) dy$ , where  $C$  is the unit circle, parameterized in counterclockwise direction. (Don't even try to solve this integral directly.)