

Final Preparation Problems, Advanced Calculus, Fall 2008

1. True or false?

- (a) If $g \circ f$ is one-to-one, then f is one-to-one.
- (b) If $g \circ f$ is one-to-one, then g is one-to-one.
- (c) If (a_n) is a bounded sequence, then $b_n = \frac{a_1 + \dots + a_n}{n}$ converges.
- (d) If $b_n = \frac{a_1 + \dots + a_n}{n}$ converges, then (a_n) is bounded.
- (e) Every continuous function $f : [0, +\infty) \rightarrow \mathbb{R}$ is bounded.
- (f) Every continuous function $f : [0, +\infty) \rightarrow \mathbb{R}$ with $\lim_{x \rightarrow \infty} f(x) = 0$ is bounded.

2. Find the limit of these sequences or show that it does not exist.

- (a) $a_n = \frac{n}{\sqrt{n+1}}$
- (b) $b_n = \frac{3^n - (-2)^n}{3^n + (-2)^n}$
- (c) $c_n = \frac{2^n + (-3)^n}{2^n - 3^n}$
- (d) $d_1 = 0$, and $d_{n+1} = d_n^2 + 1/4$ for $n \geq 1$.
- (e) $e_1 = 1$, and $e_{n+1} = e_n^2 + 1/4$ for $n \geq 1$.

3. Find the limits or show that they do not exist.

- (a) $\lim_{x \rightarrow \infty} \frac{1+x^2}{x^3-x^2}$
- (b) $\lim_{x \rightarrow 1} \frac{1+x^2}{x^3-x^2}$
- (c) $\lim_{x \rightarrow 0} \frac{1+x^2}{x^3-x^2}$

4. Where are the following functions continuous?

- (a) $f(x) = [x]$
- (b) $g(x) = x$ for $x \in \mathbb{Q}$, and $g(x) = 1/x$ for $x \notin \mathbb{Q}$.

5. (a) Show that the equation $r^x + x = 0$ has exactly one real solution x for every $r > 0$.

(b)* Denoting this solution by $x(r)$, show that this is a continuous function of r .