Sample Final, MATH 225, Spring 2008

This is a collection of problems to prepare for the final exam. If you can do all of these, you should be well on your way to a good grade. However, do not assume that the final exam itself will look exactly like this.

1. Determine the order of the differential equations, and state whether they are linear or nonlinear.

(a) $t^2y'' - y = \sin t$,

Second order linear.

(b) $yy' = 1$.

First order nonlinear.

2. Find the general solution of $y' = (1+t)y^2$.

$$y(t) = -\frac{1}{t + t^2/2 + C}.$$

3. Find the general solution of $ty' = 2y - 3$.

$$y(t) = \frac{3}{2} + Ct^2$$

4. (a) Draw the phase line and classify the equilibrium points for $y' = 4 - y^2$.

Equilibrium points are 2 (stable) and -2 (unstable).

(b) Draw the direction field and sketch several solutions of $y' = 4 - y^2$. What happens as $t \to \infty$, depending on the initial value $y(0) = y_0$?

If $y_0 < -2$, then $y(t) \to -\infty$; if $y_0 = -2$, then $y(t) \to -2$; if $y_0 > -2$, then $y(t) \to 2$.

5. One of the following two differential equations is exact. Find out which one it is, and find the general solution for it.

(a) $(2 + y\sin xy)dx + (1 + x\sin xy)dy = 0$

Exact, solution is $2x + y - \cos xy = C$.

(b) $(2 + x\sin xy)dx + (1 + y\sin xy)dy = 0$

Not exact.

6. (a) Find the general solution of $y''' + 3y'' + 3y' - 7y = 0$.

$$y(t) = c_1e^t + c_2e^{-2t}\cos(\sqrt{3}t) + c_3e^{-2t}\sin(\sqrt{3}t)$$

(b) Find one solution of $y''' + 3y'' + 3y' - 7y = t^2 - 2e^t$.

$$y(t) = -\frac{t}{6}e^t - \frac{1}{7}t^2 - \frac{6}{49}t - \frac{60}{343}$$

7. (a) Find the general solution of $y'' + 2y' + 2y = 0$.

$$y(t) = c_1 e^{-t}\cos t + c_2 e^{-t}\sin t$$

(b) Find one solution of $y'' + 2y' + 2y = e^{-t}\sec t$.

$$y(t) = e^{-t}\left((\cos t)\ln|\cos t| + t\sin t\right)$$

8. Find the first four terms in the power series solution of the initial value problem $y'' + \dfrac{y}{1-x} = 0$, $y(0) = 1$, $y'(0) = 0$.

$$y(x) = 1 - \frac{1}{2}x^2 - \frac{1}{6}x^3 + \dots$$

9. (a) Find the Laplace transform of

$$g(t) = \begin{cases} 1, & t < 1 \\ e^{2-2t}, & t \geq 1 \end{cases}$$

$$G(s) = \frac{1}{s} + e^{-s}\left(\frac{1}{s+2} - \frac{1}{s}\right)$$

(b) Solve the initial value problem $y'' + 2y' + y = g(t)$, $y(0) = y'(0) = 0$, where $g(t)$ is the function from part (a).

$$y(t) = 1 - e^{-t} - te^{-t} + u_1(t)\left(-1 + 2(t-1)e^{1-t} + e^{2-2t}\right))$$

10. Find the general solution of the system of differential equations

$$x_1' = x_1 - 2x_2$$
$$x_2' = x_1 + 4x_2$$

$$\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = c_1 e^{2t}\begin{pmatrix} 2 \\ -1 \end{pmatrix} + c_2 e^{3t}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 2c_1e^{2t} + c_2e^{3t} \\ -c_1e^{2t} - c_2e^{3t} \end{pmatrix}$$