Second Practice Test, M221-01, Fall 2010

1. True or false? Justify your answers.

(a) The column space of AB is always contained in the column space of A.

(b) The row space of AB is always contained in the row space of A.

(c) The nullspace of a 3 by 4 matrix always has dimension at least 1.

(d) The left nullspace of a 3 by 4 matrix always has dimension at least 1.

2. Let A be a matrix and R its row reduced form. Which of the four associated subspaces (column space, row space, nullspace and left nullspace) are the same for A and R? Justify your answer.

3. If A is a 7 by 3 matrix, and its left nullspace has dimension two, find the dimensions of the other three associated subspaces.

4. Find the row reduced form R for

$$A = \begin{bmatrix} -1 & 2 & 0 & -1 \\ 2 & -3 & 1 & 0 \\ 1 & -1 & 1 & -1 \end{bmatrix}.$$

5. For the same matrix as in 4, find the general solution to $A\mathbf{x} = \mathbf{0}$.

6. For the same matrix as in 4, find the complete solution to $A\mathbf{x} = (0, 1, 1)$.

7. With the same matrix as in 4, for which vectors $\mathbf{b}$ does $A\mathbf{x} = \mathbf{b}$ have a solution?

8. With the same matrix as in 4, find bases for the column space, row space, nullspace, and left nullspace of A, as well as for its reduced row form R