Final Review Problems, M 273, Fall 2011

1. Show that the equation $\cot\phi = 2\cos\theta + \sin\theta$ in spherical coordinates describes a plane through the origin, and find a normal vector to this plane.

2. Find the length of the path $\mathbf{r}(t) = \langle \sin 2t, \cos 2t, 3t - 1\rangle$ for $1 \le t \le 3$.

3. A force $\mathbf{F} = \langle 12t + 4, 8 - 24t\rangle$ (Newtons) acts on a 2-kg mass. Find the position of the mass at $t = 2$ (seconds) if it is located at $(4, 6)$ at $t = 0$ and has initial velocity $\langle 2, 3\rangle$ (m/s).

4. Find an equation of the tangent plane at $P = (0, 3, -1)$ to the surface with equation $ze^x + e^{z+1} = xy + y - 3$.

5. Find the minimum and maximum values of $f(x, y, z) = x - z$ on the intersection of the cylinders $x^2 + y^2 = 1$ and $x^2 + z^2 = 1$.

6. Find the double integral of $f(x, y) = x^3y$ over the region between the curves $y = x^2$ and $y = x(1 - x)$.

7. Use cylindrical coordinates to find the mass of the solid bounded by $z = 8 - x^2 - y^2$ and $z = x^2 + y^2$, assuming a mass density of $f(x, y, z) = (x^2 + y^2)^{1/2}$.

8. Calculate the work required to move an object from $P = (1, 1, 1)$ to $Q = (3, -4, -2)$ against the force field $\mathbf{F}(x, y, z) = -12r^{-4}\langle x, y, z\rangle$ (distance in meters, force in Newtons), where $r = \sqrt{x^2 + y^2 + z^2}$. Hint: Find a potential function for $\mathbf{F}$.

9. Find the flow rate of a fluid with velocity field $\mathbf{v} = \langle 2x, y, xy\rangle$ m/s across the part of the cylinder $x^2 + y^2 = 9$ where $x \ge 0$, $y \ge 0$, and $0 \le z \le 4$ (distance in meters).

10. Use Green's Theorem to evaluate $\oint_{\mathcal{C}} xy\,dy - y^2\,dx$, where $\mathcal{C}$ is the unit circle in counterclockwise orientation.