

## First Test Review Problems, M 273, Fall 2011

1. Let  $\mathbf{v} = \langle 1, 3, -2 \rangle$  and  $\mathbf{w} = \langle 2, -1, 4 \rangle$ .
  - (a) Compute  $\mathbf{v} \cdot \mathbf{w}$ .
  - (b) Compute the angle between  $\mathbf{v}$  and  $\mathbf{w}$ .
  - (c) Compute  $\mathbf{v} \times \mathbf{w}$ .
  - (d) Find the area of the parallelogram spanned by  $\mathbf{v}$  and  $\mathbf{w}$ .
  - (e) Find the volume of the parallelepiped spanned by  $\mathbf{v}$ ,  $\mathbf{w}$ , and  $\mathbf{u} = \langle 1, 2, 6 \rangle$ .
  - (f) Find all vectors orthogonal to both  $\mathbf{v}$  and  $\mathbf{w}$ .
  - (g) Find the decomposition  $\mathbf{v} = \mathbf{v}_{\parallel} + \mathbf{v}_{\perp}$  with respect to  $\mathbf{w}$ .
2. Find an equation for the plane through the points  $(1, 2, 3)$ ,  $(2, 3, 4)$ , and  $(3, 4, 6)$ .
3. (a) Write the spherical equation  $\rho^2(1 + A \cos^2 \phi) = 1$  in rectangular coordinates.  
(b) How does the surface depend on the constant  $A$ ?
4. Find a parametrization of the intersection of the plane  $x + y + z = 1$  and the elliptical cylinder  $(\frac{y}{3})^2 + (\frac{z}{8})^2 = 1$ .
5. A force  $\mathbf{F} = \langle 12t + 4, 8 - 24t \rangle$  (in Newtons) acts on a 2 kg mass. Find the position of the mass at  $t = 2$ s if it is located at  $(4, 6)$  at  $t = 0$  and has initial velocity  $\langle 2, 3 \rangle$  in m/s.
6. Let  $\mathbf{r}(t) = \langle \sin 3t, 4t, \cos 3t \rangle$  for  $0 \leq t \leq 2\pi$ .
  - (a) Find  $\mathbf{r}'(t)$ ,  $\mathbf{r}''(t)$ ,  $\mathbf{T}(t)$ , and  $\mathbf{N}(t)$ .
  - (b) Find an arc length parametrization of the path.
  - (c) Find the curvature of the path.