

Third Test Review, M273Q-03, Spring 2011

1. Calculate $\iint_D (1+x^2) dA$, where D is the triangular region with vertices $(0,0)$, $(1,1)$, and $(0,1)$.

2. Change the order of integration and evaluate

$$\int_0^9 \int_0^{\sqrt{y}} \frac{x dx dy}{(x^2 + y)^{1/2}}.$$

3. Find the centroid of the region \mathcal{W} bounded in spherical coordinates by $\phi = \phi_0$ and the sphere $\rho = R$.

4. (a) Parametrize the circle \mathcal{C} of radius 2 with center $(4,5)$ in counterclockwise orientation.

(b) Find $\oint_{\mathcal{C}} (x+y) ds$.

(c) What could a possible physical interpretation of the integral in (b) be? Give one example. (There are many correct answers here.)

5. One of the following vector fields is conservative. Find a potential for it, and use the potential to calculate $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$, where the curve \mathcal{C} is given by $\mathbf{r}(t) = \langle t^{3/2}, \cos(\pi t^2) \rangle$, $0 \leq t \leq 1$.

$$\mathbf{F}_1(x, y) = \langle ye^{xy} + y, xe^{xy} - x \rangle$$

$$\mathbf{F}_2(x, y) = \langle ye^{xy} - x, xe^{xy} + y \rangle$$

6. Calculate

$$\iint_{\mathcal{S}} (x^2 + y^2)e^{-z} dS,$$

where \mathcal{S} is the cylinder with equation $x^2 + y^2 = 9$ for $0 \leq z \leq 10$.