Final Review Problems, M 273, Spring 2011

1. True or false? Correct the false statements.

(a) If two vectors $\mathbf{a}$ and $\mathbf{b}$ are parallel, then $\mathbf{a} \cdot \mathbf{b} = 0$.

(b) The curve with vector equation $\mathbf{r}(t) = \langle 2t^3, -t^3, 3t^3 \rangle$ is a line.

(c) If $\mathbf{u}$ is a unit vector perpendicular to $\nabla f(\mathbf{x}_0)$, then the directional derivative $D_{\mathbf{u}} f(\mathbf{x}_0)$ is zero.

(d) $\displaystyle\int_0^2 \int_0^{\sqrt{4-x^2}} \frac{1}{x^2+y^2}\, dy\, dx = \int_0^2 \int_0^{\pi/2} \frac{1}{r}\, d\theta\, dr.$

(e) If C is a circle of radius R, then $\displaystyle\oint_C x\, dy = R^2$.

2. Find the area of the triangle with vertices $(-1, 0, 1)$, $(0, 0, 3)$, and $(-2, 1, 1)$

3. Identify and sketch the surface $x^2 = y^2 + 4z^2 - 4z$.

4. An athlete throws a ball at an angle of $30°$ to the horizontal at an initial speed of 30 m/s. It leaves his hand 2 meters above the ground.

(a) Find the velocity $\mathbf{v}(t)$ and the position $\mathbf{r}(t)$ at time t.

(b) How high does the ball go?

(Assume that the only relevant force is gravity with an acceleration of $\approx 10\,\mathrm{m/s^2}$.)

5. Find the tangent plane to the surface $z = \dfrac{6}{1+x^2+y^2}$ at the point $(1, 2, 1)$.

6. Find and classify the critical points of $f(x, y) = x^3 - y^3 + 3xy$.

7. Find the maximum and minimum values of $f(x, y, z) = 8x - 4z$ on the ellipsoid $x^2 + 10y^2 + z^2 = 5$.

8. Consider a triangular lamina D with vertices $(0, 0)$, $(1, 0)$, and $(0, 1)$, with mass density $\rho(x, y) = 1 + x + y$. Find its total mass.

9. Evaluate $\displaystyle\int_0^2 \int_0^{\sqrt{4-z^2}} \int_0^{\sqrt{4-z^2-y^2}} z\, dx\, dy\, dz$ by changing to spherical coordinates.

10. Let the curve C be given by $\mathbf{r}(t) = \langle \cos 2t, t, \sin 2t \rangle$, $0 \le t \le \pi$. Sketch the curve and find $\displaystyle\int_C y\, ds$.

11. Find a potential for $\mathbf{F}(x, y) = \langle 2 + y\cos(xy), 3 + x\cos(xy) \rangle$ and use it to evaluate $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the upper half of the unit circle $x^2 + y^2 = 1$, in counterclockwise direction.