First Test Review Key, M 273, Fall 2011

1. Which of the points $(0,1,2)$, $(3,4,0)$, $(-2,0,-3)$, $(1,-1,1)$ is closest to the xy-plane? Which point is in the xz-plane?

SOLUTION. Closest to the xy-plane is $(3,4,0)$, in the xz-plane is $(-2,0,-3)$.

2. A wagon is pulled a distance of 50 m by a constant force of 20 N. The handle of the wagon is held at an angle of $45°$. How much work is done?

SOLUTION. $W = 50 \cdot 20 \cdot \cos 45° = \frac{1000}{\sqrt{2}} \approx 707\,\mathrm{J}$.

3. Which of the following statements are true, which are false?

(i) $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$

(ii) $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{a}$

(iii) $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u} = \mathbf{0}$

(iv) $\frac{d}{dt}(\mathbf{u}(t) \cdot \mathbf{v}(t)) = \mathbf{u}'(t) \cdot \mathbf{v}'(t)$

(v) $\frac{d}{dt}(\mathbf{u}(t) \times \mathbf{u}(t)) = \mathbf{0}$

SOLUTION. True are (i), (iii), (v), false are (ii), (iv).

4. (a) Write down an equation for the plane which contains the points $(1,2,3)$, $(2,3,4)$, and $(3,4,6)$.

(b) Which of the points $(0,1,2)$ and $(0,2,1)$ lies in this plane?

(c) Find the normal vector of the plane $y + z = 3$.

(d) Find parametric equations for the line of intersection of the planes in (a) and (c).

SOLUTION. (a) $x - y = -1$; (b) $(0,1,2)$; (c) $\langle 0,1,1\rangle$; (d) $x = -t$, $y = 1 - t$, $z = 2 + t$.

5. Reduce the equation $x^2 - 2x + 2y = 2z^2$ to one of the standard forms, classify the surface and (try to) sketch it.

SOLUTION. Standard form is $\frac{y-2}{2} = 2z^2 - (x-1)^2$. It is a hyperbolic paraboloid (saddle) with the center at $(1,2,0)$.

6. A river flowing east is 10m wide, and the water speed in the river is given by the function $f(x) = \frac{1}{5}x(10-x)$ (in m/s), where x is the distance from the north bank in meters. A boat proceeds with a constant speed of 2 m/s (relative to the water) from a point A on the north bank, heading straight south. How far down the river will the boat arrive on the south bank?

SOLUTION. $50/3 \approx 16.67$ m.

7. Consider the space curve given by $\mathbf{r}(t) = \langle \cos 2t, \sin 2t, t^2\rangle$.

(a) Find the unit tangent vector at $t = \pi$.

(b) Find the limit of the unit tangent vector as $t \to \infty$.

(c) Find the length of the curve between $t = 0$ and $t = \pi$.

SOLUTION. (a) $\mathbf{T}(1) = \frac{1}{\sqrt{1+4\pi^2}}\langle 0,1,2\pi\rangle$.

(b) $\mathbf{T}(t) = \frac{1}{\sqrt{4+4t^2}}\langle -2\sin 2t, 2\cos 2t, 2t\rangle \to \langle 0,0,1\rangle$ as $t \to \infty$.

(c) $L = \int_0^\pi \sqrt{4+4t^2}\,dt = \pi\sqrt{1+\pi^2} + \ln(\pi + \sqrt{1+\pi^2}) \approx 12.2$.