First Test Review Problems, M 273, Fall 2011

1. Which of the points $(0, 1, 2)$, $(3, 4, 0)$, $(-2, 0, -3)$, $(1, -1, 1)$ is closest to the xy-plane? Which point is in the xz-plane?

2. A wagon is pulled a distance of 50 m by a constant force of 20 N. The handle of the wagon is held at an angle of $45°$. How much work is done?

3. Which of the following statements are true, which are false?

(i) $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$
(ii) $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{a}$
(iii) $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u} = \mathbf{0}$
(iv) $\frac{d}{dt}(\mathbf{u}(t) \cdot \mathbf{v}(t)) = \mathbf{u}'(t) \cdot \mathbf{v}'(t)$
(v) $\frac{d}{dt}(\mathbf{u}(t) \times \mathbf{u}(t)) = \mathbf{0}$

4. (a) Write down an equation for the plane which contains the points $(1, 2, 3)$, $(2, 3, 4)$, and $(3, 4, 6)$.

(b) Which of the points $(0, 1, 2)$ and $(0, 2, 1)$ lies in this plane?

(c) Find the normal vector of the plane $y + z = 3$.

(d) Find parametric equations for the line of intersection of the planes in (a) and (c).

5. Reduce the equation $x^2 - 2x + 2y = 2z^2$ to one of the standard forms, classify the surface and (try to) sketch it.

6. A river flowing east is 10m wide, and the water speed in the river is given by the function $f(x) = \frac{1}{5}x(10 - x)$ (in m/s), where x is the distance from the north bank in meters. A boat proceeds with a constant speed of 2 m/s (relative to the water) from a point A on the north bank, heading straight south. How far down the river will the boat arrive on the south bank?

7. Consider the space curve given by $\mathbf{r}(t) = \langle \cos 2t, \sin 2t, t^2 \rangle$.

(a) Find the unit tangent vector at $t = \pi$.

(b) Find the limit of the unit tangent vector as $t \to \infty$.

(c) Find the length of the curve between $t = 0$ and $t = \pi$.