Second Test Review Problems, M 273, Fall 2011

1. Find and sketch the domain of the function $f(x, y) = \ln(x - \sqrt{y})$. Sketch a few level curves of f.

2. Find the partial derivatives f_x, f_y, f_z, and f_{xyz} of $f(x, y, z) = \dfrac{xy^2 - yx^2}{z}$.

3. Find the linear approximation of $f(x, y) = x + y - \sin(x^2 - 4y^2)$ at the point $(2, 1)$, and use it to estimate $f(2.01, 0.99)$.

4. Use the chain rule to find $\partial z/\partial r$ and $\partial z/\partial \theta$ for $z = e^{-x^2-y^2}$, $x = r\cos\theta$, $y = 2r\sin\theta$ at $r = 1, \theta = 0$.

5. Find the directional derivative of $f(x, y, z) = \dfrac{x^2 - y^4}{z}$ at $(4, 2, 1)$ in the direction of $\mathbf{v} = \langle 0, 3, -4\rangle$.

6. Find all critical points of $f(x, y) = x^3 - 3x + y^2$ and determine whether they are local maxima, minima, or saddle points.

7. Find an equation for the tangent plane to $xyz = \cos(x + 2y + 3z)$ at $(2, -1, 0)$.

8. True or false? (Assume that f has continuous second partial derivatives.)

(a) If $\nabla f(x, y) = \mathbf{0}$, then f has a local extremum at (x, y).

(b) If f has a local extremum at (x, y), then $\nabla f(x, y) = \mathbf{0}$.

(c) If f has two local maxima, then f must have a local minimum.

(d) $f_{xy} = f_{yx}$.