Third Test Review Problems, M273, Spring 2011

1. Find $\iiint_E y^2 z^2 \, dV$, where E is bounded by the paraboloid $x = 1 - y^2 - z^2$ and the plane $x = 0$.

2. Sketch the region of integration, reverse the order of integration and evaluate $\int_0^4 \int_{\sqrt{y}}^2 \frac{y e^{x^2}}{x^3} \, dx \, dy$.

3. Consider a lamina that occupies the region D between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ in the first quadrant with mass density equal to the distance to $(0, 0)$. Find the mass and center of mass of the lamina.

4. Calculate $\int_C y \, ds$, where C is the part of the graph $y = 2x^3$ from $(0, 0)$ to $(1, 2)$.

5. Find the work done by the force field $\mathbf{F}(x, y) = \langle y, -x \rangle$ on a particle that moves along the graph of $y = x^3 - x$ from $(-1, 0)$ to $(1, 0)$.

6. Which of the following vector fields are conservative? Find a potential for one of them and use it to calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the arc of the unit circle from $(1, 0)$ to $(0, 1)$ in counterclockwise direction.

$$\mathbf{F}_1(x, y) = \langle x^2, x^2 \rangle \qquad \mathbf{F}_2(x, y) = \langle 2xy, x^2 \rangle$$
$$\mathbf{F}_3(x, y) = \langle e^y, e^x \rangle \qquad \mathbf{F}_4(x, y) = \langle e^x, e^y \rangle$$

7. Use Green's Theorem to evaluate $\oint_C \sin(1 + x^2) dx + x(1 + y) \, dy$, where C is the unit circle.