Fall 2016 – M 597

Topics in Mathematics **Homework 2**

Complex Dynamics Due Friday, September 16, in class

1. Let g be a non-constant meromorphic function in the domain $U \subset \hat{\mathbb{C}}$, let $V = g(U)$, and let $\mathcal{F}$ be a family of meromorphic functions on V. Show that $\mathcal{F}$ is normal on V iff the family of compositions $\{f \circ g \colon f \in \mathcal{F}\}$ is normal in U.

2. Let $\mathcal{F}$ be a family of meromorphic functions on a domain D, and let $\epsilon > 0$ such that each $f \in \mathcal{F}$ omits three values $w_1, w_2, w_3 \in \hat{\mathbb{C}}$ (which may depend on f) such that $d(w_j, w_k) \geq \epsilon$ for $j \neq k$. Show that $\mathcal{F}$ is normal. (Hint: Consider the Möbius transformation T_f which maps (w_1, w_2, w_3) to $(0, 1, \infty)$, apply Montel's Theorem to the family $\{T_f \circ f \colon f \in \mathcal{F}\}$, and, given any sequence $\{f_n\}$, use the ϵ-separation condition to show that $\{T_{f_n}^{-1}\}$ always has a converging subsequence.)

3. Let $\{f_n\}$ be a sequence of rational functions such that $f_n \to f$ locally uniformly on $\hat{\mathbb{C}}$. Show that f is rational, and that there exists n_0 such that f has the same degree as f_n for $n \geq n_0$.

4. Let f be a rational map of degree at least 2. Show that the Julia set J_f can not be empty. (Hint: Use the previous problem.)