Fall 2016 – M 597

Topics in Mathematics **Homework 3**

Complex Dynamics Due Wednesday, September 28, in class

1. Let f be an analytic map with a fixed point at ∞. Show that the multiplier of f at ∞ is equal to $\lambda = \lim\limits_{z\to\infty} \frac{1}{f'(z)} = \lim\limits_{z\to\infty} \frac{z}{f(z)}$.

2. Let f be a non-constant rational map and let $K \subseteq \hat{\mathbb{C}}$ be a set.

   (a) Show that $f^{-1}(K) = K$ implies $f(K) = K$.

   (b) Give an example to show that $f(K) = K$ does not imply $f^{-1}(K) = K$.

   (In other words, for complete invariance it is enough to check $f^{-1}(K) = K$, but not enough to check $f(K) = K$.)

3. Let f be a rational map of degree ≥ 2. Show that the Julia set $\mathcal{J}(f)$ is the smallest completely invariant compact set containing at least three points. I.e., if K is a compact set containing at least three points with $f(K) = K = f^{-1}(K)$, then $\mathcal{J}(f) \subseteq K$.

4. Let $f(z) = z^2 - 2$. Show that $\mathcal{J}(f) = [-2, 2]$.