M 597: Complex Dynamics, Fall 2016

Class MWF 10:00–10:50 am, 2-244 Wilson Hall (Hurst Conference Room)

Instructor Lukas Geyer, 2-254 Wilson, office hours TBD (check website), or by appointment, Tel. 994-5342, email `geyer@montana.edu`

Class Website `http://www.math.montana.edu/geyer/2016-fall-m597/index.html`

Main Reference John Milnor, *Dynamics in one complex variable: Introductory lectures*, Princeton University Press.

Other References See website.

Prerequisites A graduate Complex Analysis course or consent of the instructor.

This is an introductory graduate-level course on one-dimensional complex dynamics. The main goal is to understand the dynamical behavior of iterated rational maps on the Riemann sphere. Topics covered will include

- Normal Families
- Julia and Fatou sets
- Fixed points and periodic points
- Classification of Fatou components
- Parameter spaces, Mandelbrot set
- Quasiconformal surgery, renormalization

Complex Dynamics as a field was developed in the early 20th century in some groundbreaking papers by Pierre Fatou and Gaston Julia. It became very popular both in the public eye as well as in the mathematical community in the 1980s, mainly through the advance of computer graphics, which made it possible for the first time ever to accurately picture fractal Julia sets and the Mandelbrot set. At the same time, Dennis Sullivan introduced quasiconformal maps as a very powerful new tool into the theory, revolutionizing the mathematical techniques and leading to many new results since.

This class will cover the classical theory in the first part of the course, and at least part of the modern theory in the later part of the semester, with a particular emphasis on quasiconformal surgery and renormalization. Based on student and instructor interest, the content of the second part is very flexible and will be determined during the semester.

The ambitious goal is to get to interesting material related to current research, so I will review and introduce (mostly without proofs) several results from Complex Analysis which are typically not covered in an introductory one-semester class. In the second half, I will give an informal introduction into the theory of quasiconformal mappings.

Milnor's book is listed as the main reference, and it is a great introduction to the theory. However, there are some topics that Milnor does not cover, or that are covered more thoroughly in other sources. There are also some topics in Milnor's book which will not be covered in this class. In the later part of the course we will probably cover material that has not yet found its way into textbooks. Since this class will not follow a single text, the most important requirement of the students is to *come to class*, and try to get notes and catch up in case a class is missed. The grades for the course will be based on participation in class (for all students) and on selected homework assignments and/or class presentations (for those students who are not yet working on a Ph. D. thesis.)