

**Correcting for Time-of-Day to Compare
Satellite Images and Ground Counts of
Weddell Seals in Big Razorback Haul-Out
(Erebus Bay, Antarctica)**

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This writing project has been read by the writing project advisor and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the Statistics Faculty.

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Abstract

This project is aimed to improve and expand upon an ongoing study of Weddell Seal (*Leptonychotes weddellii*) populations along the Antarctic coast. Ten populations of seals living in Erebus Bay have been studied for the last 50 years. The proximity of McMurdo Station to Erebus bay allows scientists to access these seals for observation. The focus of this project concerns only the seals that use the Big Razorback (BR) site as a breeding ground. Satellite images, ground counts, and ground camera data are available from BR for the year 2010. Although the satellite images are capable of identifying seals on the ice, they were obtained at different times of the day and different days of the year than the ground counts. Therefore, these satellite counts need to be adjusted before being compared to ground counts. The objective of this project is to develop a method to accurately count adult seals using satellite imagery alone. The use of such technology includes benefits for both the ecologists and the seals alike as it is non-invasive and does not require the use of field personnel. A potential implication of this method would be the ability to estimate abundances for seal populations inaccessible to humans. To evaluate whether this is feasible we have to account for several discrepancies between the satellite imagery and the ground count data available. The satellite images were not taken at the same time or on the same day as the censuses on the ground, so there is no ground truth for the satellite photos. However, the camera data provides an image of a portion of BR every 45 minutes for 52 consecutive days during the seals breeding season (October 18, 2010 - December 08, 2010). The adults and pups from these images were carefully counted to enumerate seals present on the ice throughout each day for the portion of BR captured in the images. Assuming the behavior of the seals is consistent across BR, the camera data can be used to build a model relating time of day to the abundance of seals on the ice. A correction factor for the time and day discrepancies between satellite and ground counts will be estimated from this model as well as paired uncertainty intervals. This correction factor will adjust the satellite counts so they can be more directly compared to the ground counts. Our analysis could not account for all discrepancies between the satellite and ground counts, but is successful for certain satellite photos. We are hopeful that with a designed experiment we could greatly improve this method and these results.

1 Introduction

The Weddell Seal project began in 1968 with researchers studying seals at ten sites in Erebus Bay, Antarctica. The Weddell seal is one of the top predators in the Ross Sea, so studying their population dynamics provides a wealth of information about the Antarctic marine ecosystem as a whole (Rotella J., 2012). The primary interest of researchers is the relationship between seal population dynamics and temporal variation in the climate which affects sea-ice conditions in the Ross Sea (Weddell Seal Science, 2012). Knowledge about the abundance of seals at each site is critical information for understanding these relationships. This is increasingly important in these times of climate change.

Weddell seals can be found in Erebus bay during their reproductive season, which is in the Antarctic spring between the months of October and December. These seals can hold their breath for up to fifty minutes, which allows them to dive deep under the fast ice to tide cracks where they haul out for breeding season in a virtually predator free environment. This makes the species unique in the sense that their pups have a high rate of survival just after birth. A benefit of this environment for the researchers is that the seals allow humans to approach and study them while they are with their newborn pups. The seals in Erebus Bay have been part of an ongoing mark-recapture study funded by the National Science Foundation (NSF) for the last 44 years (Garrott, R. and Rotella, J., 2012).

Much is known about the Weddell seal populations in the ten haul-out sites in Erebus Bay, but Weddell seals can be found anywhere along the antarctic coast where tide cracks in the fast ice exist (LaRue, M. and Rotella, J. et. al, 2011). This poses an extremely limiting logistical constraint to researchers: the Antarctic coast is large and there are many areas, inaccessible to humans, that may host populations of Weddell

seals. In order to understand the species as a whole it is necessary to study more than just the populations of seals in Erebus Bay. Thus, a recent development in the Weddell Seal Project has been facilitating the use of satellite imagery to count seals. The long-term goal of using satellite imagery is to develop a way to count populations of seals that cannot be accessed by researchers (LaRue, M. and Rotella, J. et. al, 2011). However, before this is a realistic endeavor, it must be established that satellite photos give an accurate representation of seals on the ice. To establish this, satellite image counts must be compared to ground counts from areas accessible to the researchers. The focus of this project will be to determine whether satellite imagery can be used to accurately count seals at the Big Razorback haul-out site using data collected during the 2010 breeding season

Past research has shown that satellite counts provide an underestimate of the actual number of seals on the ground, but it is believed this may largely be due to a time discrepancy between the two different methods (LaRue, M. and Rotella, J. et. al, 2011). Satellite images are taken around 10:00am and ground counts are collected between the hours of 3:00pm and 6:00pm, which makes for an inequitable comparison. The most seals are on the ice between 12:00pm and 7:00pm (LaRue, M. and Rotella, J. et al, 2011). Thus satellite images are taken when many seals are in the water and the ground counts are taken when many seals are on the ice. To account for this discrepancy, the researchers needed a way to relate time of day to the number of seals on the ice at the BR site. With this goal in mind, they set up a digital camera that took a picture of a region of BR every forty-five minutes for fifty two days in 2010 (October 18 - December 8). These camera images provided counts of adult seals throughout each day for a subset of the BR site. With the assumption that seals behave the same throughout the site, this data could be used to model adult seal

count by time of day (Figure 1a).

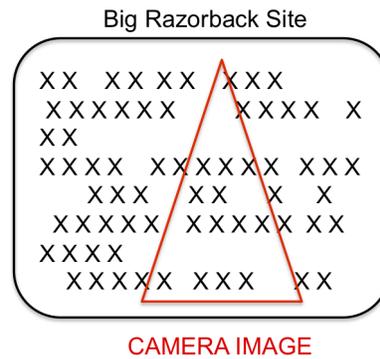


Figure 1a: *The camera images capture only a small portion of the entire BR site. The X's represent seals, the triangular border represents the view of the camera, and the rectangular border represents the entire field site (The figure is not to scale).*

The researchers felt it was safe to assume that seal behavior was consistent across BR, so the counts from the camera images could be used to quantify a relationship between the number of seals on the ice and time of day that could be assumed to apply across over the entire BR site.

2 Methods

The objective is to build a regression model that will allow us to approximate a correction factor for the time discrepancy between the ground counts and the satellite image counts (Figure 1b).

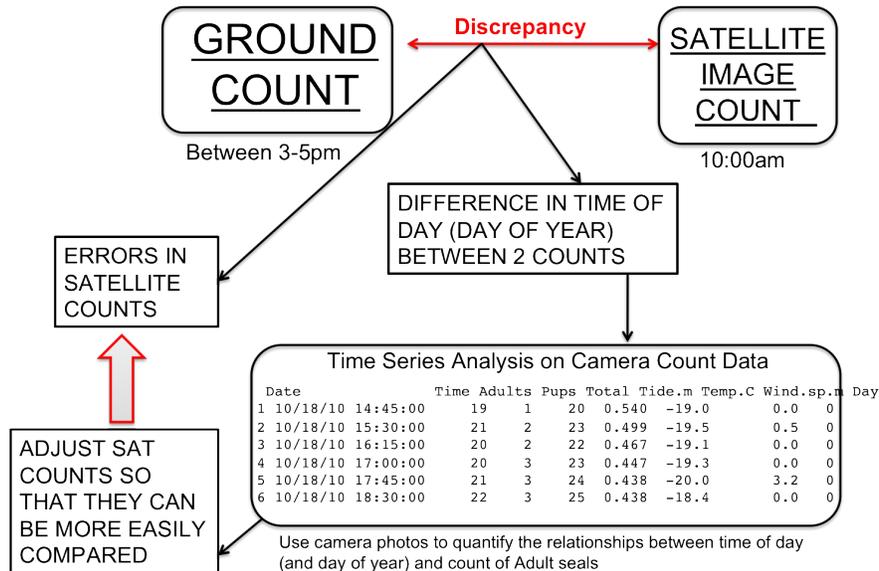


Figure 1b: *The camera count data will be used to build a regression model which will use Adult seal counts as the response and time of day, day of year, temperature, tide, and wind speed as explanatory variables. A correction factor for the satellite photos will be estimated based on this model to calibrate satellite photos with actual ground counts.*

First, a reasonable model will be built for the relationship between the number of adult seals on the ice, time of day, and day of year using the camera count data from 2010. This model will be used to estimate a correction factor for discrepancies in counts due to time of day. The quantification of these discrepancies will be the correction factors for each satellite image. A standard error for these correction factors will be estimated and approximate 95% confidence intervals will be computed. To evaluate the accuracy of the estimated correction factor, these intervals will be multiplied by the satellite image counts and compared to ground counts.

2.1 Data

Three types of data are available from the year 2010: ground counts of BR from six days between October 18 and December 8, nine satellite images of BR that were taken on six days between October 18 and December 8, and camera data that provides counts of seals on the ice in a region of BR every 45 minutes for the entire 52 days. The satellite images are very good at detecting adult seals, but not as good at detecting the pups due to their small size (LaRue, M. and Rotella, J. et. al, 2011). Because of this, the response variable of interest from the camera count data will be counts of adults. Figure 2 shows scatterplots of adult seal counts and natural log (\ln) transformed counts from the camera images for all days in the study.

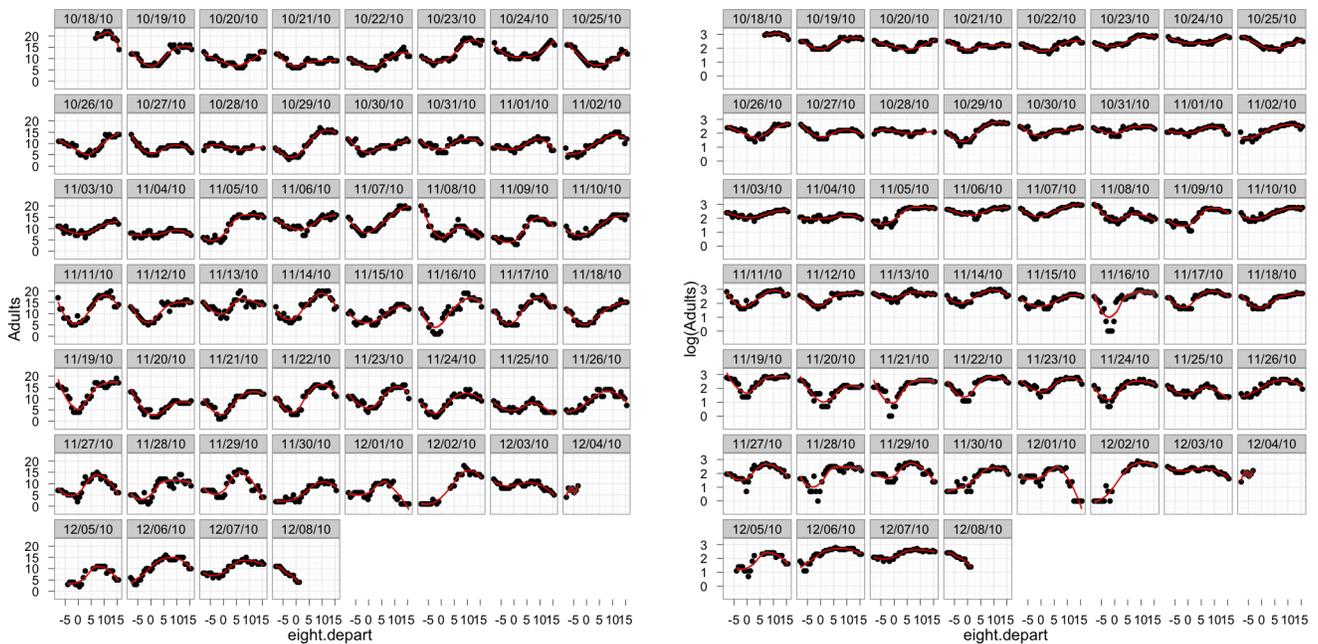


Figure 2: *Counts of adult seals vs. time of day (measured in hours from 8:00am) (left). Log transformation of adult seal counts vs. eight.depart (right). As expected, there appears to be a cyclical pattern each day, where the minimum number of seals on the ice occurs near 10:00 am and the maximum near 5:00pm*

Figure 2 illustrates the problem the time discrepancy poses for estimating abundance of seals using satellite imagery. In these plots time of day (TOD) is the x-

variable and is referenced from 8:00am (TOD = 1 indicates an observation taken at 9:00am). Around 10:00am, when the satellite photos were taken (2 on the x-axis of the scatterplot) the counts are near the minimum number of seals on the ice. Around 5:00pm (9 on the x-axis) most seals are present on the ice. Therefore, the satellite photos and the ground counts represent observations during low and high seal presence respectively. Another discrepancy between the satellite photos and ground counts is that the images do not come from the same days as the ground counts (Table 1). Therefore, in order to adjust the satellite image counts appropriately we must match satellite days with ground count days.

Table 1: *Dates, times and counts for the ground and Satellite images. Because ground counts were not instantaneous like the satellite images we used the time in the middle of the duration of the ground count as TOD.*

Date	Type	Count	Start	End	Duration
6-Nov-10	Ground	130	2:58 PM	7:58 PM	4:59 hrs
11-Nov-10	Ground	131	3:08 PM	6:49 PM	3:41 hrs
17-Nov-10	Ground	131	10:04 AM	2:45 PM	4:41 hrs
21-Nov-10	Ground	87	10:25 AM	12:27 PM	2:02 hrs
25-Nov-10	Ground	121	2:36 PM	5:07 PM	2:31 hrs
3-Dec-10	Ground	103	11:39 AM	1:06 PM	1:21 hrs
Date	Type	Count	Time		
30-Oct-10	Sat	43	9:06 AM		
30-Oct-10	Sat	64	10:23 AM		
31-Oct-10	Sat	76	10:01 AM		
31-Oct-10	Sat	80	10:09 AM		
5-Nov-10	Sat	57	8:48 AM		
5-Nov-10	Sat	51	9:46 AM		
8-Nov-10	Sat	79	10:19 AM		
27-Nov-10	Sat	116	10:05 AM		
7-Dec-10	Sat	163	10:31 AM		

2.2 Models

To quantify the relationship between counts of Adult seals on the ice and time of day, three types of models were considered: an ordinary least squares regression model, an ordinary least squares regression using a log transformed response (multiplicative least squares model), and a Poisson log-linear regression model. It is known that variables such as tides (meters), temperature ($^{\circ}\text{C}$) and wind speed (meters/sec)

affect the behavior of the seals (whether or not they leave the ice to fish and swim with their pups) and thus counts of seals on the ice. Therefore, measurements for these variables were recorded each time an image was taken. These covariates and time of day will be used as predictors to model the number of seals on the ice in the camera count images. The estimation of a correction factor will depend entirely on the model chosen to explain seal abundance.

An additive ordinary least squares (OLS) model would result in a correction factor that adds some number of seals to the satellite photo count to calibrate them with the ground counts. Although this may work if we had camera images that encapsulated the entire area, it makes more sense to develop a correction factor that would bump up the satellite counts by some multiplicative factor to account for the time discrepancy between the two measurements. To achieve this, we fit a model using a natural log transformation of the response variable (count of adults) (Equation 1). A correction factor from this model multiplies the number of seals observed by the satellite image so that it is comparable to the ground count obtained from a later time of day. In the camera count data, there were no observations where the counts of the Adults were 0, so the data could be transformed without adjusting any of the observations.

$$\log(Adults_i) = \beta_0 + \beta_1 TOD_i + \beta_2 TOD_i^2 + \beta_3 TOD_i^3 + \beta_4 Tide_i + \beta_5 Temp_i + \beta_6 Wind_i + \epsilon_i \quad (1)$$

$$\text{Where } \epsilon_i \stackrel{iid}{\sim} N(0, \sigma_{\log(Adults)}^2) \quad i = 1, 2, \dots, 1583$$

Count data are integer valued and therefore can often be modeled using the Poisson distribution within a generalized linear model (Equation 2). Poisson log-linear regression assumes the variance of the residuals is equal to the mean and that the residuals are independent. Often in practice the variance of residuals increases as the estimated mean increases when dealing with count data. When this occurs, there is

over-dispersion in the data and the Poisson model will estimate standard errors that are too small. These liberal estimates of standard errors can lead to confidence intervals that are too narrow, which leads to false precision (Gelman A. and Hill J., 2007). The estimated over-dispersion parameter from the model assuming independence was 1.020, suggesting the Poisson distribution may be useful for modeling these data. Like the least squares multiplicative model, the parameters from a Poisson log-linear regression are estimated on the log scale so the correction factor from this model would be multiplicative.

$$Adults_i \stackrel{iid}{\sim} Poisson(\theta_i) \quad (2)$$

$$\log(\theta_i) = \beta_0 + \beta_1 TOD_i + \beta_2 TOD_i^2 + \beta_3 TOD_i^3 + \beta_4 Tide_i + \beta_5 Temp_i + \beta_6 Wind_i \quad i = 1, 2, \dots, 1583$$

The objective of this study was to estimate correction factors for satellite images taken on specific days in 2010, which are dependent on the model used to relate counts of seals on the ice and time of day. We are most interested in explaining the behavior of seals on days satellite images were obtained and less concerned with their average behavior throughout the 2010 breeding season. Both model (1) and (2) assume the same curve for each day, which resulted in a poor job capturing the low and high points of seals on the ice for most days. We need to capture the difference between the high and low counts on particular days to accurately estimate a correction factor, so this is a problem. A comparison of the three regression fits is shown in Figure 3. All three models yield similar results.

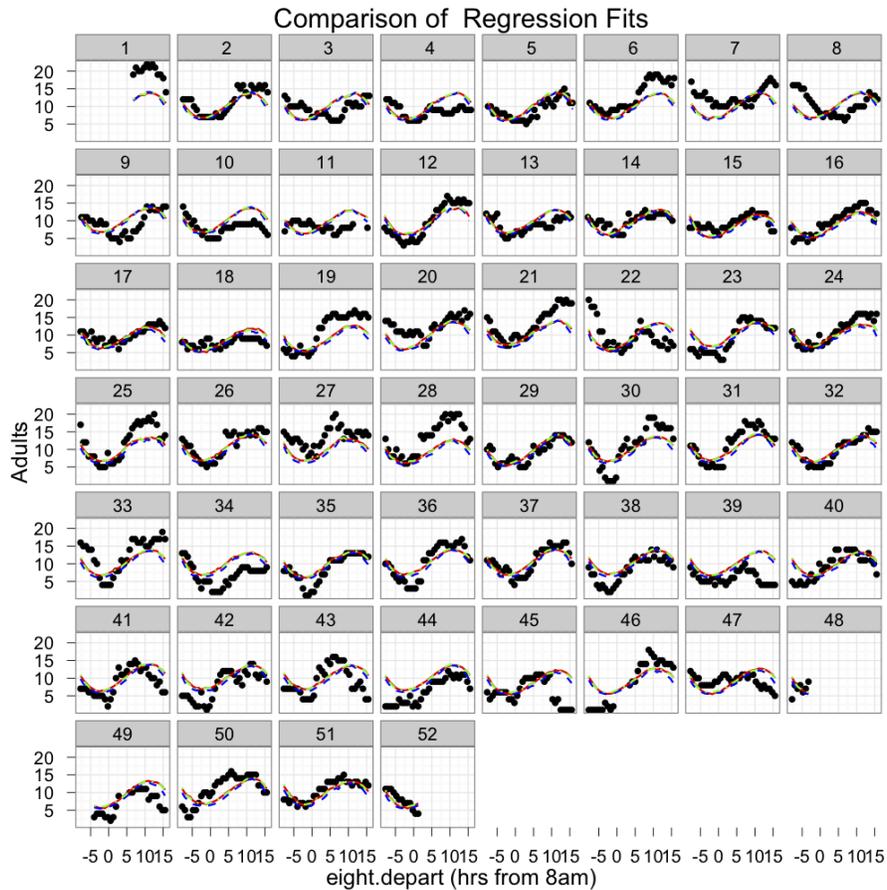


Figure 3: Comparison of regression fits on the original scale. Red is the additive model, blue is the multiplicative model, and green is the Poisson log-linear regression.

To more appropriately explain how many seals are on the ice at any point in a day in 2010, day was included as a factor. This modeled a separate curve for each day. We treated day as a fixed effect because we were interested in specific days during 2010. The result was a model that captured the seal behavior during the peak and low haul out times much more accurately. Separate curves for each day were fit for both the log transformed least squares regression and the Poisson log-linear regression. A comparison of both regressions is shown in Figure 4.

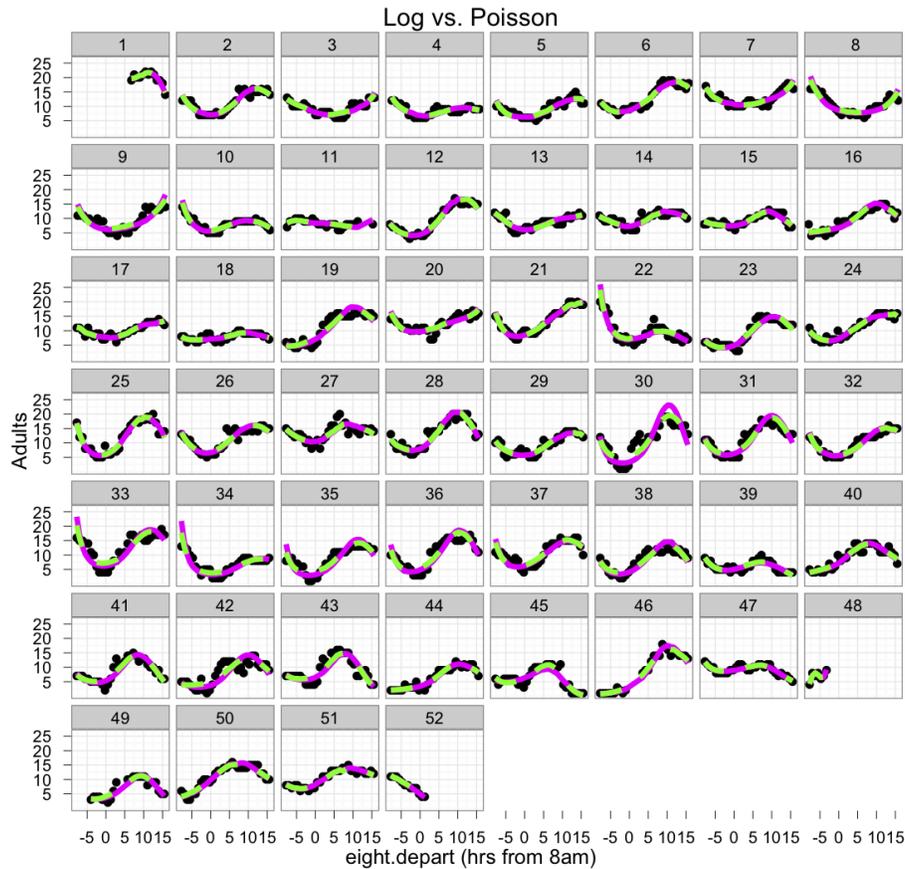


Figure 4: The magenta line is the fitted curve from the log-transformed least squares regression model (Equation 1) and the green line is the fitted curve for the Poisson log-linear regression model (Equation 2). Both models allowed for a different line for each day.

2.3 Checking Assumptions

Visually, the regression lines from both models are similar. Quantitatively, the standard errors of the Poisson log-linear model are slightly larger than those for the log-transformed model and the coefficient estimates were slightly smaller. We assessed whether the models adhered to the underlying assumptions that were used to build them as a source for comparison. Both models assume the same relationship between seal counts and the tide temperature and windspeed for all 52 days, which seems appropriate.

In the log transformed regression, we assumed the errors were distributed normally

and that there was constant variance. Figure 5a shows diagnostic plots for these assumptions. There appears to be more variability in lower $\log(\text{counts})$ of seals than in higher $\log(\text{counts})$ of seals. This indicates a relationship between the fitted values (the mean) and the variability, though opposite of what one would expect. The Normal quantile plot indicates that the assumption of normally distributed errors for this model is violated in the tails. Although there are many large residuals, the residuals vs. leverage plot shows no indication that any of these are particularly influential. Because there are no high leverage points and we have a sample size of 1583, we can appeal to Gnedenko and Kolmogorov's Central Limit Theorem for linear models (Robison-Cox J., 2012).

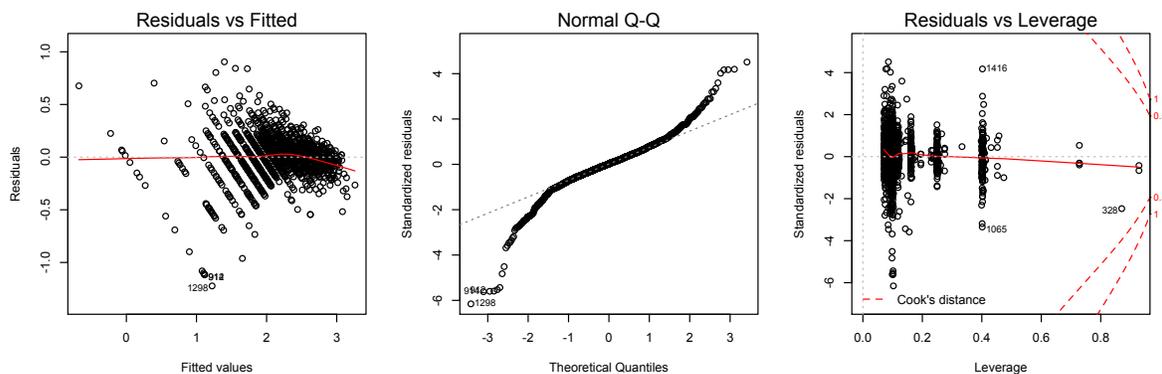


Figure 5a: Diagnostic plots from the log-transformed regression model (first four) and Poisson log-linear regression model (last two)

The Poisson log-linear regression assumed constant variance and that the residuals have variance equal to their mean. When this is violated, the data are said to be over-dispersed (they are more variable than expected). The estimated over-dispersion for the Poisson log-linear regression that included day as a factor was 0.25903, which indicates under-dispersion. The results of under-dispersion are conservative estimates of standard errors, which will give less precise confidence intervals for our correction factor, but practically speaking are not of much concern (Gelman A. and Hill J., 2007).

The Poisson log-linear regression also assumes constant variance. Diagnostic plots for this assumption are shown in Figure 5b. There is a small improvement from the log transformed least squares regression, but still some evidence of heteroskedasticity of variance.

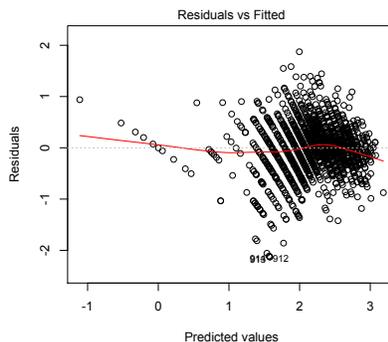


Figure 5b: Diagnostic plots from the log-transformed regression model (first four) and Poisson log-linear regression model (last two)

Both models assume their errors are independent. However, the data we are using are counts of seals at multiple times of day throughout 3 months in a year. The errors are clearly not independent. This can be seen in plots of the residuals for any given day. Figure 6 shows plots from six days.

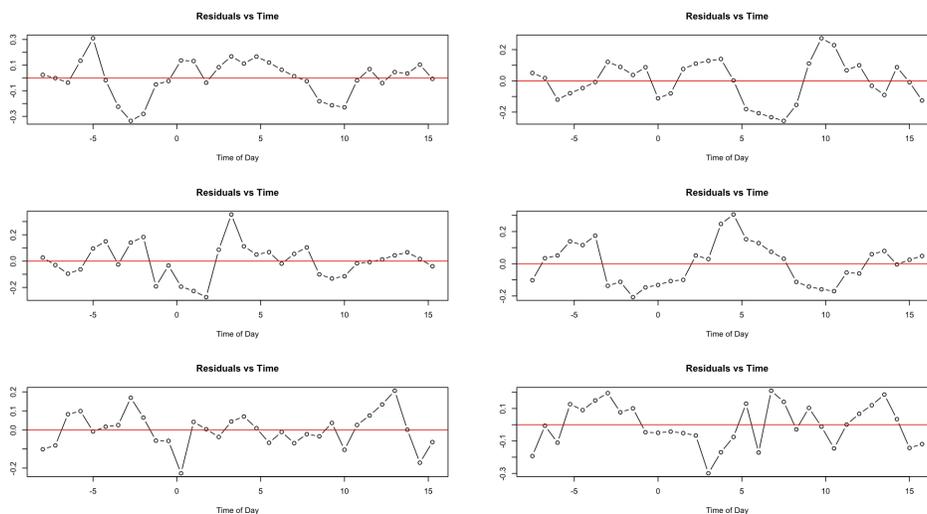


Figure 6: Residuals vs. time from the log transformed LS regression for six different days. There are distinct runs away from the overall mean where many residuals are positive and/or negative, indicating positive autocorrelation in the data.

Figure 6 shows evidence of positive autocorrelation in the residuals. This makes the data appear to be less variable than they are because measurements close in time are close to each other. To account for this, a correlation structure must be implemented in both models (Ramsey, F. and Schafer D., 2002). This is important in the context of our problem because we want to assess whether corrected satellite counts can be used to count seals on the ice. If we leave correlation out of the residuals when it is present, we will build confidence intervals that are too narrow and may conclude that the corrected satellite counts are not comparable to the ground counts when in fact they are.

After accounting for serial correlation in the residuals, the usual assumptions are not severely violated for either method, meaning that inference from both models may be valid. Our goal was to build correction factors for both models and leave the decision of which to use to the discretion of the researcher. However, we were not confident the results from the Poisson log-linear regression including a correlation structure were appropriate, so a further investigation will be part of our future work.

2.4 Time Series Analysis

The camera data are comprised of counts of seals on the ice every 45 minutes for 52 days, so the residuals are clearly not independent. The observations in our data set are equally spaced (45 mins apart) throughout the 52 days, so an autoregressive (AR) structure may be appropriate. Autocorrelation and partial autocorrelation plots for each day individually, as well as the 52 days as a long time series were made (Figures 7, 8). Exploratory residual analyses for both regression models were conducted and yielded similar results. The results and figures in this section were produced using residuals from the log-transformed least squares regression model.

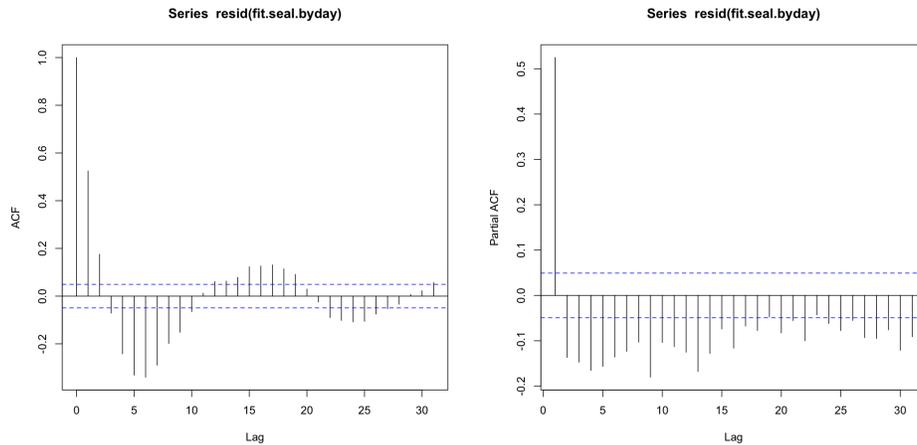


Figure 7: *ACF (left) and PACF (right) plots for the 52 days as one long time series*

Both of the plots in Figure 7 are indicative of a non-stationary time series because the autocorrelation and the partial autocorrelation of the residuals persist up to large time lags. It is customary to use difference operators on non-stationary time series' to remove persistent trends (Brockwell, P. and Davis A., 2002). However, based on the residual plots by day, the negative partial autocorrelation does not make sense for these residuals, so we chose to further examine the partial autocorrelation plots for each day individually. We found that all days had similar PACF plots and show nine of these in Figure 8.

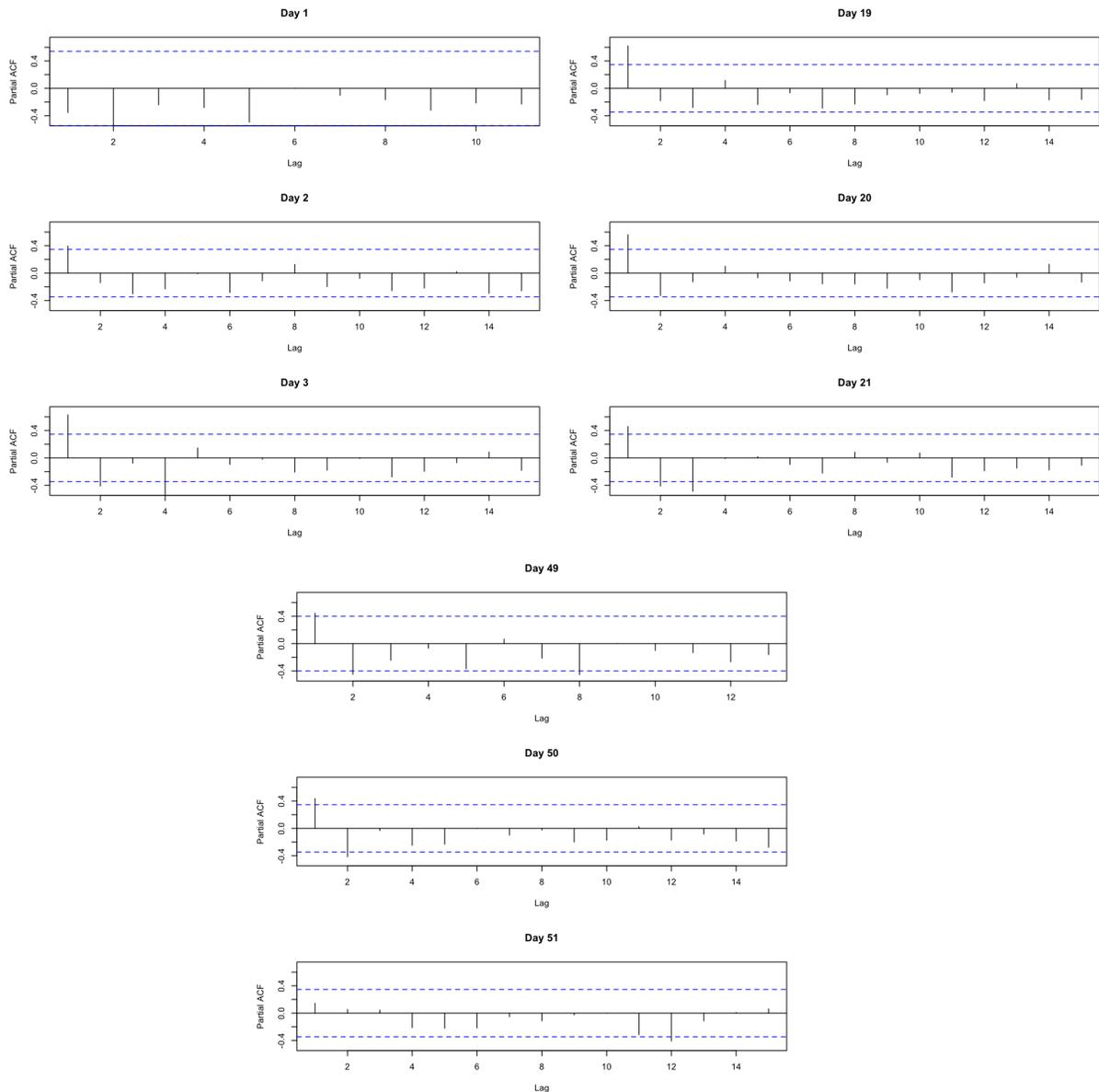


Figure 8: *Partial autocorrelation plots for 9 of the 52 days. All days are similar and most have strong positive lag 1 partial autocorrelations and little evidence of correlations at other lags after accounting for the first order serial correlation.*

With the exception of the first and last plots, which indicate no partial autocorrelation, all days have a significant lag 1 autocorrelation around 0.5. For most days, the partial autocorrelations disappear after accounting for lag 1. If we assume each correlation structure is the same for each day, it is appropriate to fit an AR(1) correlation structure. An AR(1) process defines each observation in the time series

to depend on the previous observation: Let $\{X_t\}$ be a stationary time series, then $X_t = \phi X_{t-1} + Z_t$; $t = 0, \pm 1, \dots$ and $Z_t \sim N(0, \sigma^2)$, $|\phi| < 1$. The parameter ϕ will be estimated and is the lag 1 partial autocorrelation coefficient. After removing the relationship with time of day, stationarity is assumed. This is a valid assumption because we are modeling the residuals and not the raw time series. Because $\{X_t\}$ is stationary (X_1, \dots, X_n) and $(X_{1+h}, \dots, X_{n+h})$ have identical joint distributions (Brockwell, P. and Davis A., 2002). Therefore, the covariance structure for the log-transformed least squares regression model will be of the form of Equation 3.

$$\text{Var}(\log(\text{Adults})) = \sigma_{\log(\text{adults})}^2 \begin{bmatrix} 1 & \phi & \phi^2 & \phi^3 & \phi^4 & \dots \\ \phi & 1 & \phi & \phi^2 & \phi^3 & \\ \phi^2 & \phi & 1 & \phi & \phi^2 & \\ \phi^3 & \phi^2 & \phi & 1 & \phi & \\ \phi^4 & \phi^3 & \phi^2 & \phi & 1 & \\ \vdots & & & & & \ddots \end{bmatrix} \quad (3)$$

For the previous models the covariance structure was: $\sigma_{\log(\text{adults})}^2 \mathbf{I}_{n \times n}$, which assumes independence in the residuals.

To gain insight as to where the negative partial autocorrelation appears in the residuals, progressive partial autocorrelation plots (starting with residuals from the first day only then adding each subsequent day) were made (Figure 9). The scale was kept the same for all plots to keep them comparable.

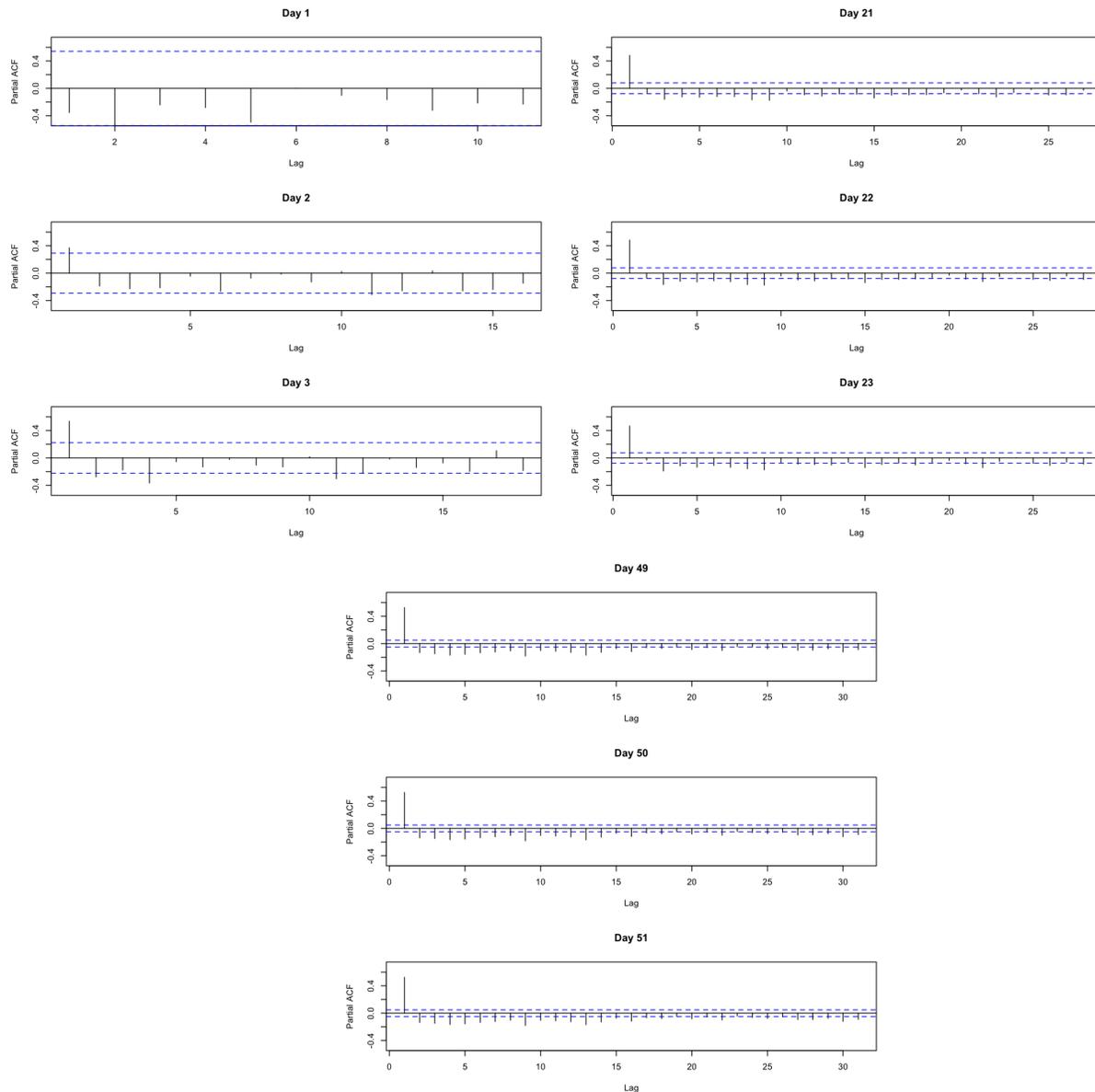


Figure 9: *Progressive partial autocorrelation plots that add subsequent days together into a single time series. The title indicates how many days' residuals have been included in the time series*

More negative partial autocorrelation gets added in each day, however, the magnitude of these negative correlations becomes smaller and smaller with each day added. There are 1583 observations in this dataset, so we have an enormous sample size. With large sample sizes we can detect trends that aren't really there, and we believe this is what is happening here. This is a classical case of practical vs. statistical sig-

nificance. Although our sample size gives us enough power to detect negative partial autocorrelations up to lag 30, they are not of practical importance. Because of this we chose to use an AR(1) correlation structure for this analysis. This was done using the `arma()` function in R with specified structure $ARIMA(1,0,0) = AR(1)$. The result of this AR(1) model was an estimated lag 1 autocorrelation of $\hat{\phi} = 0.5821$ with $SE(\hat{\phi}) = 0.0217$. This estimate is in agreement with the lag 1 autocorrelation shown on the PACF by day plots. This will bump up our standard error estimates from the independent residuals model by a factor of $\sqrt{\frac{1.5821}{.4179}} = 1.946$ (Ramsey, F. and Schafer D., 2002).

After fitting an AR(1) correlation structure for the log-transformed, ACF and PACF plots for the days were made. Figure 10 shows these plots for days on which satellite photos were obtained. The correlation between residuals for these days has been removed.

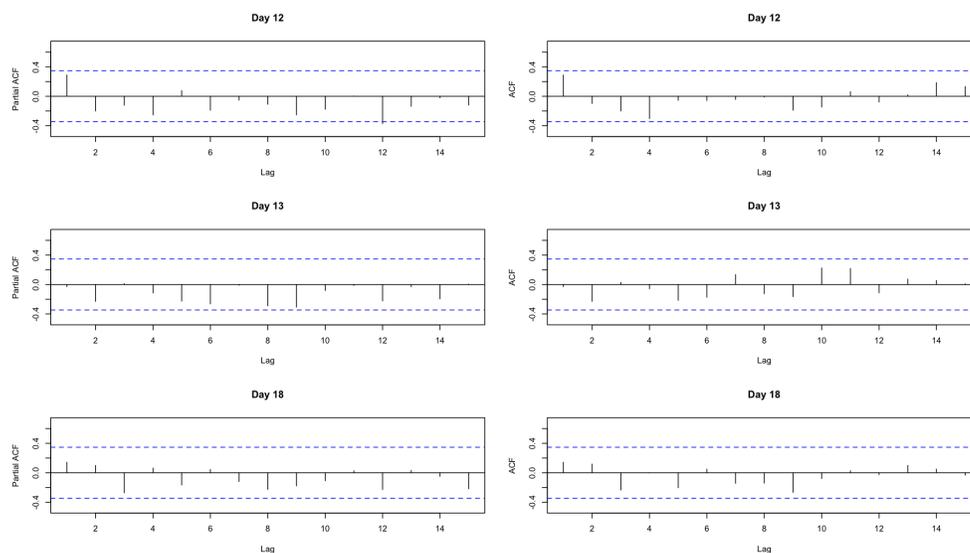


Figure 10: *ACF and PACF plots from days on which satellite photos were obtained for residuals from a log transformed response model with AR(1) correlation structure. (continued on next page)*

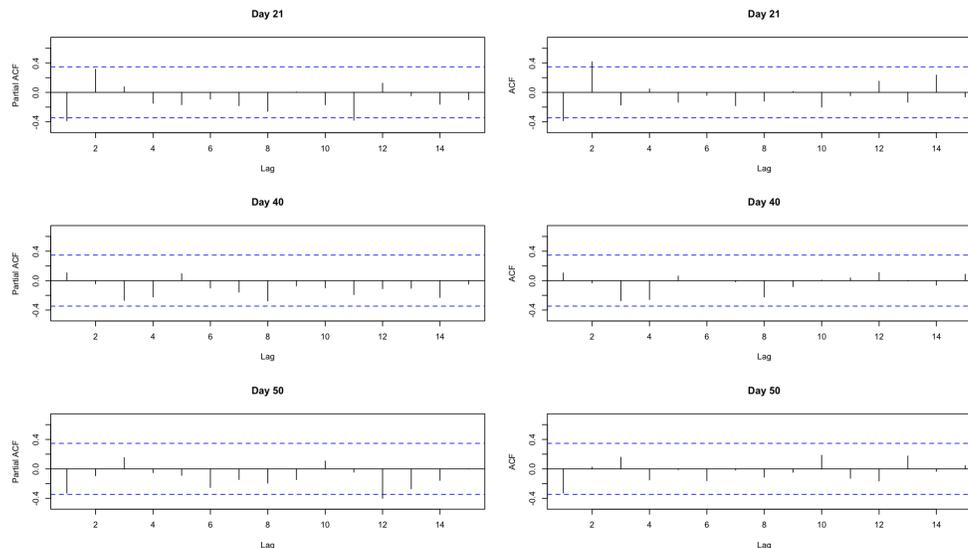


Figure 10: *ACF and PACF plots from days on which satellite photos were obtained for residuals from a log transformed response model with $AR(1)$ correlation structure*

2.5 Matching Ground Count and Satellite Image Days

Ideally, satellite images and ground counts would be available from the same days, however obtaining satellite imagery in Antarctica can be extremely difficult due to inclement weather patterns. During 2010, there were only 9 satellite images (taken on 6 days) that were viable for use.

We need to compare adjusted counts from each satellite image to appropriate ground counts. We would like to match similar ground and satellite count days. There are no natural pairings between the days from the two types of counts, so an option considered was to use the ground count day that was closest in time to the satellite count day of interest. However, we felt we could do a better job matching the seals' behaviors between the days by using a visual comparison. To do this we looked at plots of the fitted values from the log-transformed least squares regression for the days the satellite photos were taken as well as the days the ground counts were taken; these are shown in Figure 11.

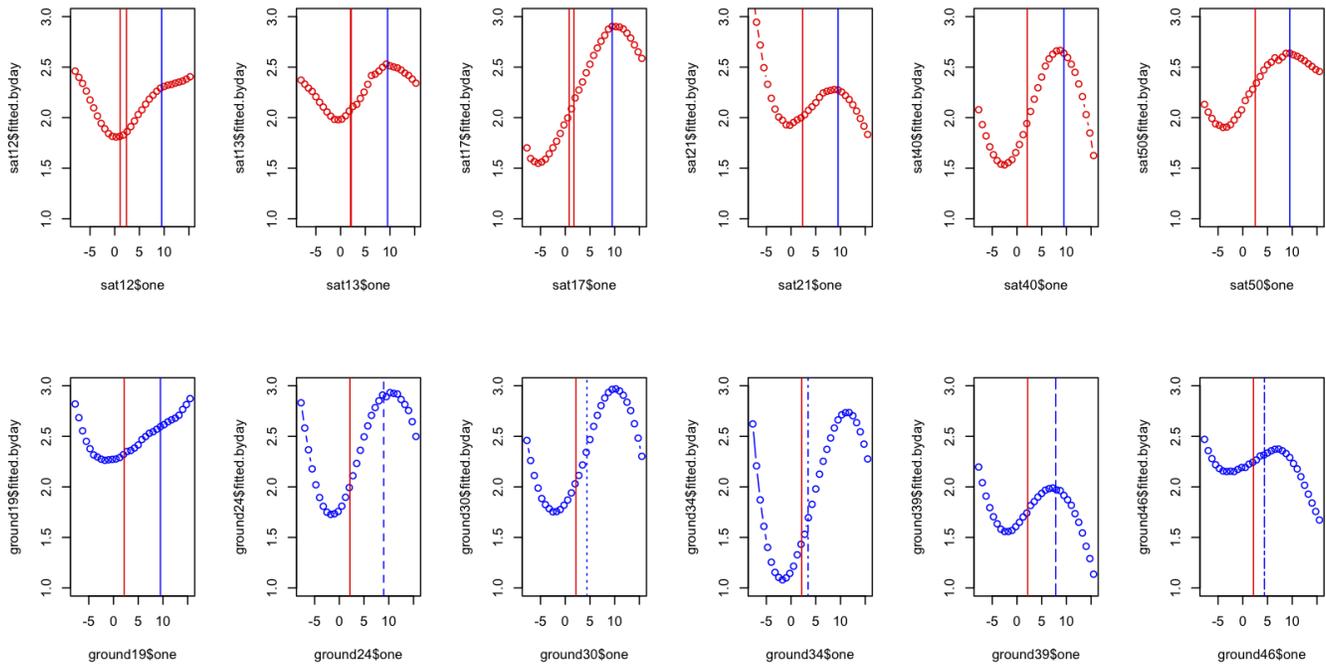


Figure 11: *Fitted values from the least squares log transformed model for counts on ground count days (bottom) and satellite image days (top). Red lines mark the times satellite photos were taken and blue lines mark the times ground counts were taken.*

The estimated correction factor is calculated as the vertical distance between the fitted value at the time the ground count was taken and the fitted value at the time the satellite image was taken. So to visually match the satellite days with ground count days, we looked for similar vertical distances across days. Figure 12 shows the results of our pairings.

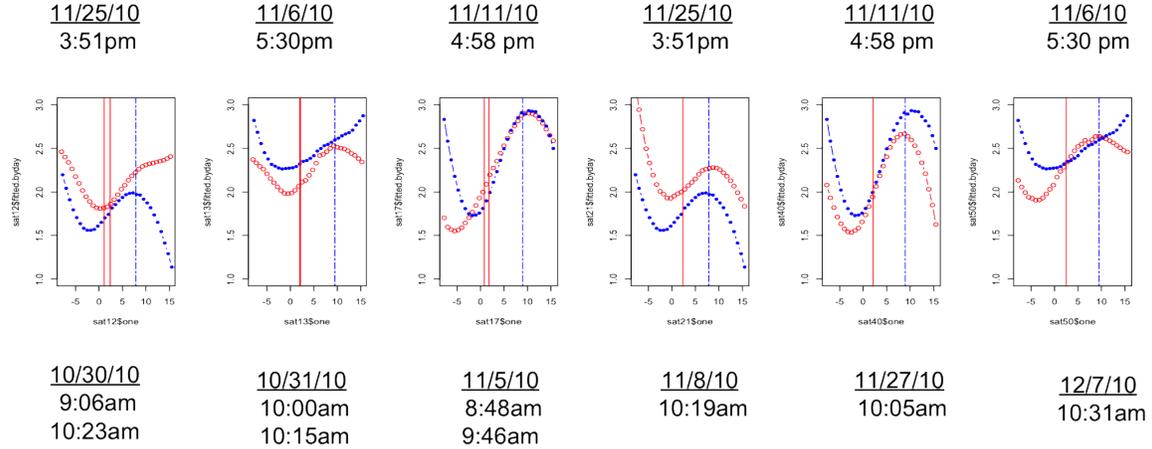


Figure 12: *Pairings: we were most concerned with matching up the vertical distances from where the red and blue lines intersect the count data on both the ground (blue points) and the satellite (red points) photos.*

2.6 Correction Factor

Two types of correction factors were built. Both used the optimal pairings between ground and satellite count days. The first correction factor assumed the ground and satellite counts were taken on the same day and used only the camera count data from the satellite day. The second method used camera count data from both the satellite photo day and the ground count day. These were both built using linear combinations of the regression coefficients. Covariate values from the time (and day (second method)) the ground and satellite counts were taken were used to build these linear combinations of regression coefficients.

Assume the ground and satellite counts were taken on the same day and let α be the true correction factor. Then the estimated correction factor, $\hat{\alpha}$, can be computed from Equation 4.

$$\hat{\alpha} = \mathbf{C}'_{\text{ground}} \hat{\beta}_{\text{Day}_{\text{sat}}} - \mathbf{C}'_{\text{satellite}} \hat{\beta}_{\text{Day}_{\text{sat}}} = (\mathbf{C}'_{\text{ground}} - \mathbf{C}'_{\text{satellite}}) \hat{\beta}_{\text{Day}_{\text{sat}}} \quad (4)$$

The standard error of this estimate is computed using Equation 5.

$$SE(\hat{\alpha}) = \sigma \sqrt{(\mathbf{C}'_{\text{ground}} - \mathbf{C}'_{\text{satellite}}) \hat{\beta}_{\text{Day}_{\text{sat}}} \mathbf{V}(\mathbf{C}'_{\text{ground}} - \mathbf{C}'_{\text{satellite}}) \hat{\beta}_{\text{Day}_{\text{sat}}}} \quad (5)$$

The variance-covariance structure of the residuals is $\sigma^2\mathbf{V}$. An approximate 95% CI for α is: $\hat{\alpha} \pm 2 * SE(\hat{\alpha})$, so a 95% CI for the corrected number of seals can be found by multiplying the back-transformed interval by the number of seals observed from the satellite photo, $\text{SatCount}_{\text{Day}_{\text{sat}}} \times e^{\hat{\alpha} \pm 2SE(\hat{\alpha})}$. These intervals will be compared to the observed ground counts on those “days” (Robison-Cox J., 2012).

When we relax the assumption that the ground count was taken on the same day we replace all $\hat{\beta}_{\text{Day}_{\text{sat}}}$ ’s with $\hat{\beta}_{\text{Day}_{\text{ground}}}$ ’s.

3 Results

For simplicity, we calculated adjusted satellite counts for the models assuming the residuals are independent. These results are discussed in sections 3.1 and 3.2. Results from a log-transformed response least squares regression including an AR(1) correlation structure for the residuals are discussed in section 3.3.

3.1 Assuming Ground Counts and Satellite Images Were Obtained from the Same Day

The adjusted satellite counts and their approximate 95% confidence intervals from both regression methods without any correlation structure are provided in Table 2.

Table 2: *Results assuming ground counts were taken on the same day as the satellite photos (log-transformed LS regression fit to the left and Poisson log-linear on the right).*

	Date	Lower	Estimate	Upper	Ground		Date	Lower	Estimate	Upper	Ground
1	10/30/10	56	66	78	121	1	10/30/10	47	62	82	121
2	10/30/10	81	93	107	121	2	10/30/10	72	90	112	121
3	10/31/10	100	119	142	130	3	10/31/10	87	113	148	130
4	10/31/10	102	121	143	130	4	10/31/10	89	116	150	130
5	11/5/10	113	135	162	131	5	11/5/10	90	118	156	131
6	11/5/10	88	104	122	131	6	11/5/10	72	93	119	131
7	11/8/10	87	100	114	121	7	11/8/10	78	97	120	121
8	11/27/10	201	244	296	131	8	11/27/10	162	217	291	131
9	12/7/10	191	223	260	130	9	12/7/10	174	220	277	130

Three of the nine intervals captured the ground count values in the log-transformed

model. Two of these occur on 10/31/10 and one on 11/5/10. The same days were successful for the Poisson log-linear model with the addition of the satellite image taken on 11/8/10. Figure 13 shows a visual representation of these results where the confidence intervals are represented by the line segments. They are ordered from smallest satellite count to largest satellite count. It's clear from Figure 13 that the standard errors for Poisson model were larger than those for the log-transformed model. For both regression models the last two intervals (corresponding to 11/27/10 and 12/7/10) are unusual in the sense that they are overestimating the ground count, which is due to the satellite counts being unusually high (116 and 163 respectively).

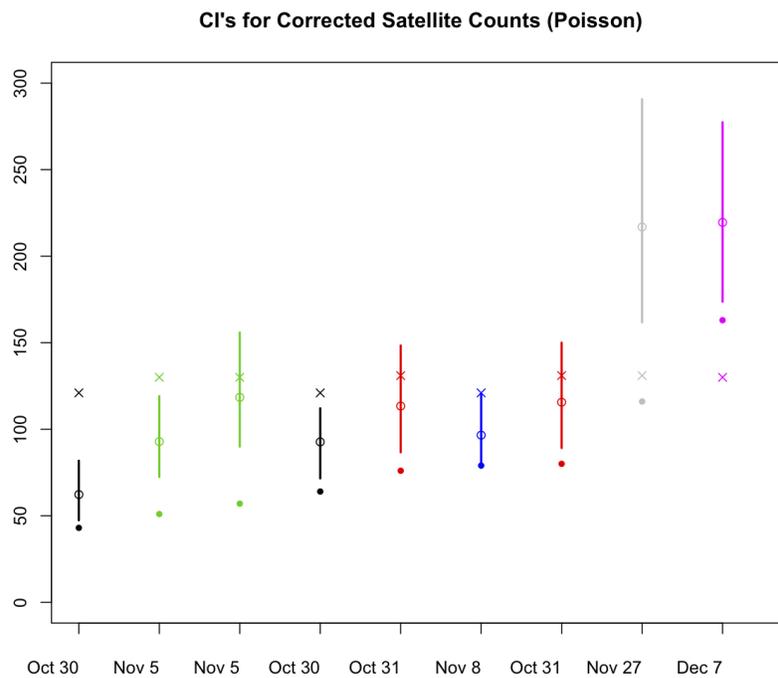
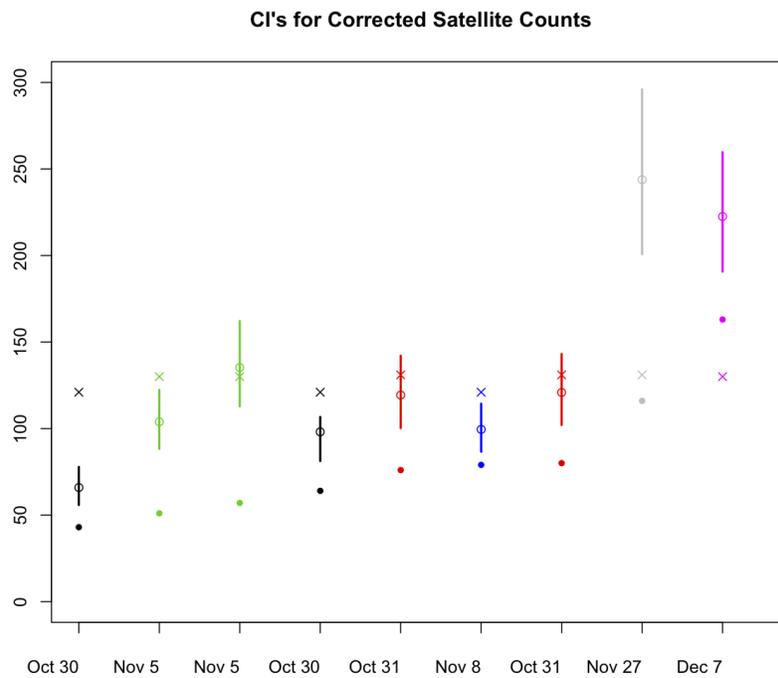


Figure 13: *CI plots from the log-transformed least squares regression model (top) and Poisson log linear model (bottom) of the adjusted satellite counts for each satellite photo day. The ×'s represent the ground count, closed points represent the number counted by the satellite, and the open circles are the point estimates using our method. Results from the Poisson log-linear model are on the next page.*

3.2 Accounting for Day Discrepancy Between Ground Counts and Satellite Images

Estimates for the adjusted satellite counts and their approximate 95% confidence intervals from both regression methods are provided in Table 3. These come from the correction factor that accounts for day.

Table 3: Results from the correction factor that accounts for day are given in Table 3. The log-transformed least squares regression is on the left and the Poisson log-linear regression is on the right.

	Date	Lower	Estimate	Upper	Ground		Date	Lower	Estimate	Upper	Ground
1	10/30/10	42	62	61	121	1	10/30/10	37	63	71	121
2	10/30/10	60	90	85	121	2	10/30/10	54	90	101	121
3	10/31/10	107	129	155	130	3	10/31/10	96	127	167	130
4	10/31/10	108	130	157	130	4	10/31/10	98	129	170	130
5	11/5/10	114	136	164	131	5	11/5/10	95	124	161	131
6	11/5/10	87	105	126	131	6	11/5/10	75	97	126	131
7	11/8/10	62	74	88	121	7	11/8/10	55	75	102	121
8	11/27/10	250	309	382	131	8	11/27/10	208	278	371	131
9	12/7/10	179	213	255	130	9	12/7/10	169	217	278	130

For both models, the only intervals that captured their respective ground counts were from both satellite images taken on 10/31/10 and one of the images taken on 11/5/10. The CI's from both models come close for the second image taken on 11/5/10, but do not capture the ground count. Rather than being bumped up, the count from the satellite image on 11/8/10 was bumped down. This was caused by the fact that the fitted value for that ground count day at the time it was recorded was actually lower than the fitted value for the satellite day at the time the satellite image was taken. This occurs with both regression fits for satellite images taken on 11/5/10, 10/30/10, and 12/7/10. As we saw with the CI's for the correction factor that assumed the ground counts were taken on the same day, the last two CI's are overestimating the ground counts. Again, this is likely due to the large counts from the images on those days (12/7/10 had a higher satellite count than ground count),

so it would be interesting if the seals' behavior has changed. A visual representation of these results is shown in Figure 14.

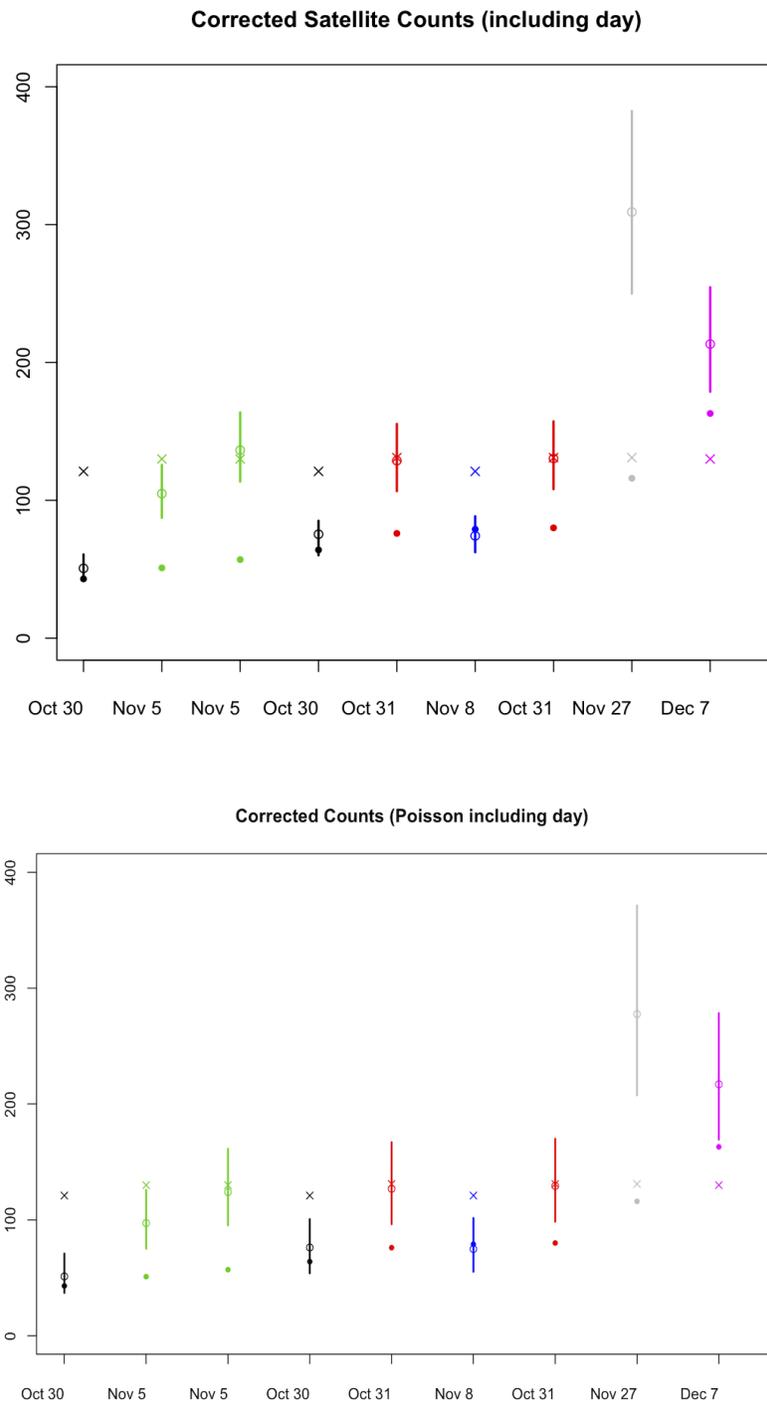


Figure 14: *CI plots for both models (log on top, Poisson on bottom) of the adjusted satellite counts for each satellite image, using both days in the linear combination of regression coefficients. The \times 's represent the ground count, closed points represent the number counted by the satellite, and the open circles are the point estimates using our method.*

3.3 AR(1) Correlation Structure

The log-transformed least squares regression model with AR(1) correlation structure produced adjusted satellite counts and corresponding approximate 95% CI's that are summarized in Table 4.

Table 4: *Results from the log-transformed LS regression model with AR(1) correlation structure. Assuming satellite and ground counts came from the same day (left) and including day in the correction factor (right)*

	Date	Lower	Estimate	Upper	Ground		Date	Lower	Estimate	Upper	Ground
1	10/30/10	51	67	88	121	1	10/30/10	37	62	68	121
2	10/30/10	76	95	118	121	2	10/30/10	52	90	95	121
3	10/31/10	96	127	167	130	3	10/31/10	99	136	187	130
4	10/31/10	99	129	168	130	4	10/31/10	101	139	190	130
5	11/5/10	100	134	179	131	5	11/5/10	99	135	186	131
6	11/5/10	83	107	140	131	6	11/5/10	80	109	148	131
7	11/8/10	75	93	115	121	7	11/8/10	53	71	96	121
8	11/27/10	176	233	308	131	8	11/27/10	216	300	417	131
9	12/7/10	171	220	282	130	9	12/7/10	158	214	291	130

When we assumed counts were obtained from the same day, both confidence intervals on 10/31/10, and one from 11/5/10 captured the ground count values. The confidence interval for the adjusted count from 11/8/10 just barely misses the ground count value for this method. However, when we include day in the construction of the correction factor, the confidence interval from 11/8/10 is nowhere near the ground count value. Rather than being bumped up, the count from the satellite image on 11/8/10 was bumped down. This again, is likely due to the fact that the fitted value on the ground count day for the time that count was taken was actually lower than the fitted value for the satellite day at the time the satellite image was taken. A visual representation of these results can be seen in Figure 15.

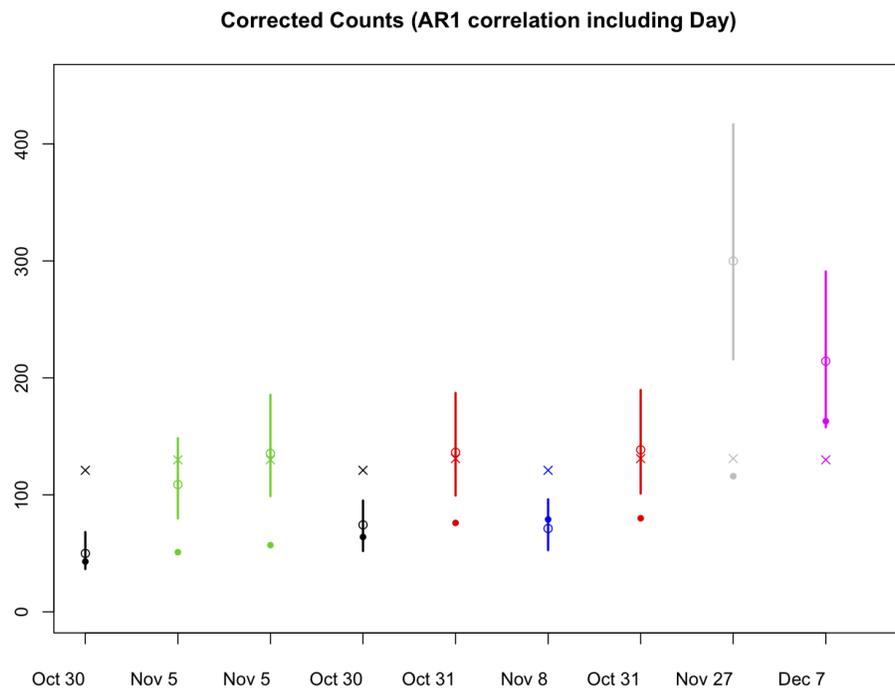
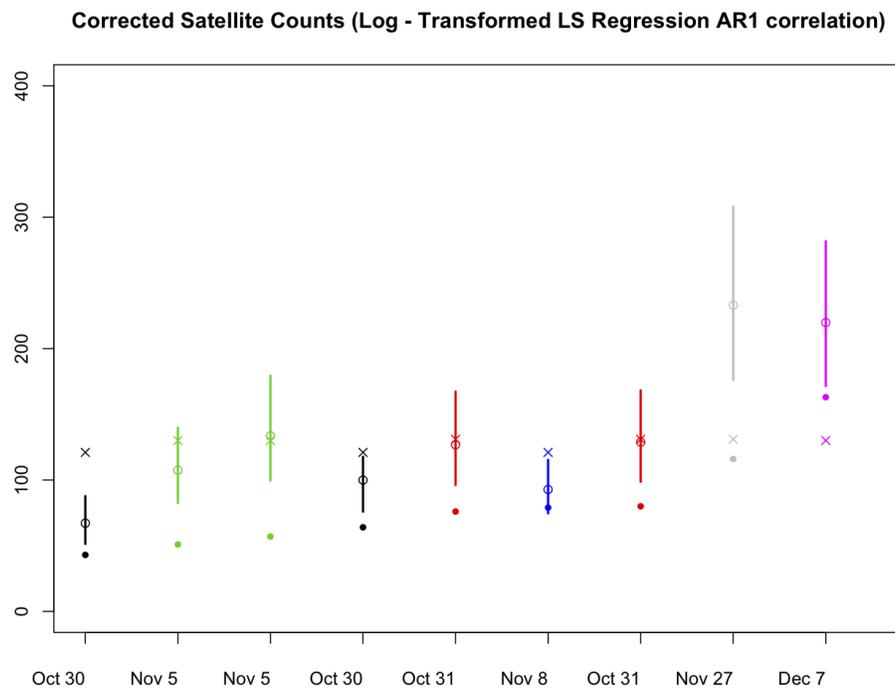


Figure 15: Confidence intervals for the adjusted satellite counts from both correction factor methods for a log-transformed response regression with $AR(1)$ correlation structure. The plot above shows CI's for the correction factor assuming counts were on the same day. The plot below shows CI's for the correction factor that included both days in its construction.

The intervals produced from the 11/27/10 and 12/7/10 satellite images overestimate the ground counts. Figure 15 shows that the satellite counts on these days are a lot higher than those on other days and that the counts for 12/7/10 exceed the ground counts before the correction factor is applied. These dates are also late in the breeding season, so this may be explained by a behavioral change in the seals towards the end of the season.

4 Future Work

Because we were only concerned with particular days in 2010 we believe an AR(1) correlation structure accounts for most of the serial autocorrelation in the residuals, however we may be able to do better with a more sophisticated correlation structure that is appropriate for the entire time series. There are ways to deal with non-stationary time series and they could be implemented for this method. Whether or not these are employed will depend on how important it is to the researches to get “wide-enough” confidence intervals.

We used a generalized estimation equation regression to incorporate an AR(1) correlation structure into the Poisson log-linear regression. This was done using the `geeglm()` function from the `{geepack}` package in R. This estimation technique assumed that observations between days were independent, so the correlations structure would be of the form of (3) for each day (Σ_j) $j = 1, 2, \dots, 52$, but the variance covariance structure for all days would have a block-diagonal structure for all observations (Equation 6) (Halekoh, Højsgaard, and June, 2006).

$$\text{Var}(\log(\text{Adults})) = \begin{bmatrix} \Sigma_1 & 0 & 0 & \cdots & 0 \\ 0 & \Sigma_2 & 0 & \cdots & 0 \\ 0 & 0 & \Sigma_3 & \cdots & 0 \\ \vdots & & & \ddots & \\ 0 & 0 & 0 & \cdots & \Sigma_{52} \end{bmatrix} \quad (6)$$

We obtained an estimated $\hat{\phi} = 0.433$ with $SE(\hat{\phi}) = 0.0289$ from this model and found similar coefficients to those from the log-transformed least squares regression method with AR(1) correlation structure, but are not confident the results are reliable. This is something we are further investigating.

5 Discussion

The results discussed in 4.1 and 4.2 assumed there was no serial autocorrelation in the camera count data, which is not the case. Because this lack of independence in the residuals goes unaccounted for, the standard errors from these models were too small in turn making the CI's too narrow. We were interested in specific days to build our correction factor from and an AR(1) correlation structure was appropriate for each day. The results from the log -transformed response least squares regression model with AR(1) correlation structure did increase the size of the confidence intervals for adjusted satellite counts, but still did not capture the ground counts with the adjusted count intervals for all days.

In general, the adjusted counts and their confidence intervals for all models over-estimated the ground counts taken on 11/27/10 and 12/7/10. The satellite counts for these days were unusually high (116 and 163) respectively. The confidence intervals for these corrected counts were also much larger than those for the other dates. Both of these dates are later in the season and we must ask: is something different going on with the seals towards the end of the pupping season? Because these dates are late in the season, the unusually large satellite counts could be due to pups being identified as adults, suggesting that false positive identifications happen late in the season.

To improve this method, satellite image days and ground count days need to be coordinated. Then only one discrepancy (time of day) is in the model. However,

logistically this may be hard to implement. Inclement weather in Antarctica makes it difficult to obtain satellite images that are viable for counts, and funding limits the number of times the researchers can physically count the seals during each pupping season. With data from only one year and mismatched satellite and ground count days, our analysis was the best possible approach.

The correction factor does what we intended for it to do when we assume satellite counts and ground counts were taken on the same days. However, it is not adjusting all counts enough to capture their corresponding ground counts. This is likely a result of the data being mismatched and could only be improved if the satellite photos were truly taken on the same day as the ground counts. This study showed promise that with the appropriate data, we may be able to accurately enumerate adult Weddell seals in BR using satellite imagery.

These data came from only one year focused on only one site, so replication is needed to assess whether the seals' behavior could be modeled in such a way across different years. Any inference concluded from these data only applies to the year 2010 at the BR site on the days satellite images were taken, which is quite restrictive. With data from more years we could determine whether the behavior of seals across years is similar and may be able to extrapolate to years other than 2010. To extrapolate beyond the BR site researchers would have to do a similar study at different sites or assume behavior of seals across all sites is the same. This project was a small step in a long journey to discover whether satellite imagery could play a role in the Weddell Seal Project in Erebus Bay, Antarctica.

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- Michelle LaRue, University of Minnesota: Department of Earth Sciences (Polar Geospatial Center Research Fellow)
- Jesse DeVoe, Montana State University: Department of Ecology

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