Finite Population Correction Methods

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1 Introduction

One application of inferential statistics is to use a simple random sample of size n from a population to learn about the characteristics of such population. A very common inference is that of the population mean. The sample mean is typically used as a point estimate for the population mean. The uncertainty in using a point estimate is addressed by means of confidence intervals. Confidence intervals provide us with a range of values for the unknown population mean along with the precision of the method. For the parametric approach, the central limit theorem allows us to construct t-based confidence intervals for large sample sizes or symmetric populations. Over the years, nonparametric computer-intensive bootstrap methods have become popular for constructing confidence intervals as well.

The central limit, the standard error of the sample mean and traditional bootstrap methods are based on the principle that samples are selected with replacement or that sampling is done without replacement from an infinite population. In most research surveys, however, sampling is done from a finite population of size N. When we sample from an infinite population or sample with replacement; then selecting one unit does not affect the probability of selecting the same or another unit. Hence, precision depends only on the sample size. With sampling from a finite population without replacement, what we see affects the probability of selection of units that we have not seen and hence, there is a finite population correction(1-n/N) arising from the lack of independence that must be accounted for in the calculation of the standard error of the sample mean.

T-based confidence intervals and ordinary bootstrap methods do not account for this lack of independence and as such these confidence intervals are approximately $\sqrt{(1-\frac{n}{N})}$ less precise with the approximation varying based on population characteristics such as skewness. In this study, I present two different nonparametric bootstrap methods for constructing confidence intervals that account for the finite population factor resulting from the lack of independence in sampling from a finite population. A simulation-based approach is used to compare the coverage rates, precision, and confidence interval widths among these two methods, the t-based approach and the ordinary bootstrap method.

2 Normal-based Confidence Interval

For a population with unknown mean μ and known standard deviation σ, a $100(1-\alpha)\%$ z-based confidence interval for the population mean is:

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

where $\bar{x}$ is the sample mean,$z_{\alpha/2}$ is the upper critical value from the standard normal distribution and $\frac{\sigma}{\sqrt{n}}$ is the standard deviation of the sample mean.

The population standard deviation σ, however, is usually unknown. If it was known, then the population mean would likely be known also. The population standard deviation σ is therefore estimated and replaced by the sample standard deviation, s in the z-based confidence interval. The substitution, however, changes the coverage probability $1-\alpha$. In 1908, Gosset from Guinness Breweries discovered the t-distribution that allow us to maintain the desired coverage level by replacing standard normal distribution critical value by the larger t-distribution critical value. The resulting t-based confidence interval for the population mean is

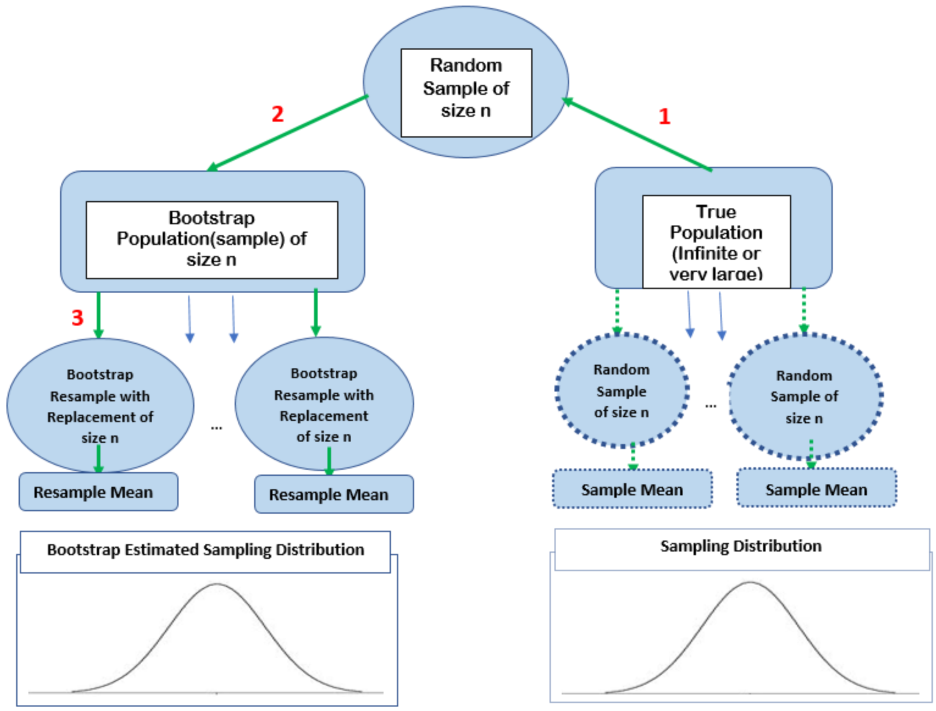
$$\bar{x} \pm t_{n-1,\alpha/2}\frac{s}{\sqrt{n}}$$

where $t_(n-1,\alpha/2)$ is the critical value determined from a t_{n-1} distribution (t distribution with n-1 degrees of freedom) and $\frac{s}{\sqrt{n}}$ is the estimated standard deviation of the sample mean called the standard error.

The confidence level is exactly $100(1-\alpha)\%$ when the population distribution is normal and is approximately $100(1-\alpha)\%$ for other distributions when the sample sizes are large by the application of the central limit theorem. With the advancement of computing powers, however, nonparametric bootstrap methods have become common in constructing confidence intervals for any sample size and for many distributions.

3 Bootstrap Confidence Interval

Bradley Efron (1979) is known to have written the first paper on the theory of bootstrapping. With subsequent papers (Efron and Tibshirani (1991), Efron and Tibshirani (1993)) and with the advancement in computing capabilities, the method gained recognition and widespread use. There are two main approaches for confidence interval estimation: parametric and the nonparametric. The former assumes that the data come from a known population distribution but with unknown parameters. Often in research survey however, the population distribution is unknown and hence, the nonparametric bootstrap helps approximate the sampling distribution of an estimator that is a function of a random sample taken from this unknown population distribution, provide standard errors for estimates and confidence intervals for the unknown parameters. The nonparametric bootstrap assumes that the population size is very large relative to the sample, which is nearly equivalent to sampling with replacement. Thus, the sample units are assumed to be independent of one another and the standard error of the sample mean, our estimator in this case, depends only on the sample size and not the fraction of the population that is sampled.



The steps in nonparametric bootstrapping are illustrated above. The dashed lines represent theoretical unobserved results.

1. A random sample of size n, $y_1, ., y_n$ is taken from a population, that is assumed to be very large or infinite.

2. A bootstrap population is created. This is typically the set of sample values $y_1 ., y_n$. The population is thought of been infinite consisting of infinite number of y_1 values, infinite number of y_2 values .infinite number of y_n values, with each occurring $\frac{1}{n}$ of the time. For example, if our sample is $(3, 20, 8, 12, 16)$ then we are assuming that 20% of the population values are $3s$, 20% are $20s$, 20% are $8s$, 20% are $12s$, and 20% are $16s$.

3. We then resample n units with replacement from the bootstrap population a total of B times. Efron and Tibshirani (1991) claimed that B=200 resamples is usually adequate to find the variance of an estimator. Practically, however, it is now computationally inexpensive to take thousands of bootstraps resamples.

4. The bootstrap sample mean is calculated from each of the B resamples, and the B bootstrap sample means are used to calculate a bootstrap standard error and generate an empirical bootstrap distribution for the sample mean.

An appropriate confidence intervals can then be found from the percentiles of the bootstrap distribution. For example, for a 95% confidence interval, we find the two values (L, U) that bound the middle 95% of the distribution. That is L and U are the 2.5th and 97.5th percentiles of the distribution. This method however, does not take into account the sampling fraction when we sample from a finite population and as such, these confidence intervals are too wide. I look at two bootstrap procedures described by A. C. Davison and D. V. Hinkley ("Bootstrap methods and their application", 1997) that account for the correlation among the sampled values.

4 Finite Population Bootstrap Sampling

4.1 Modified Resample Size.

The estimated variances of the sample mean under sampling of n units with replacement and without replacement from a finite population of size N are:

$$\left\{ \begin{array}{cc} \frac{s^2}{n} & \text{With Replacement} \\ (1-\frac{n}{n})(\frac{s^2}{n}) & \text{Replacement} \end{array} \right\}$$

where s^2 is the sample variance.
As illustrated earlier, the ordinary bootstrap involves resampling with replacement and does not consider the effect of the sampling fraction $\frac{n}{N}$ leading to larger bootstrap standard errors for the sample mean and wider confidence intervals.

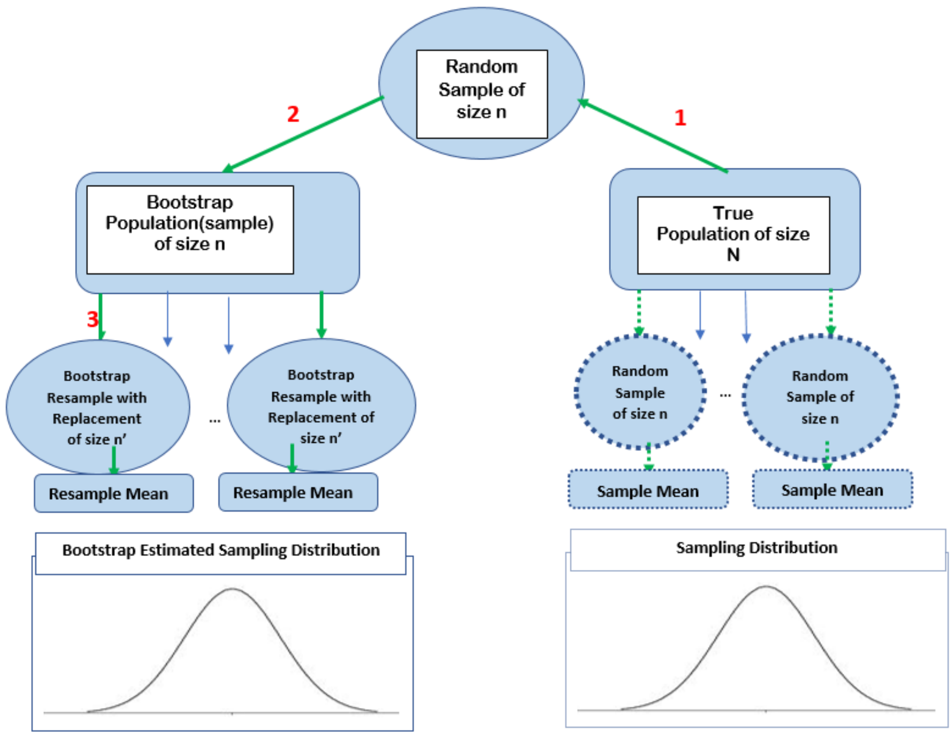
With the ordinary bootstrap with-replacement resamples, Davison and Hinkley (1997) noted that the estimated variance of the sample mean when taking resamples of size n' is

$$var(\bar{y}) = \frac{(n-1)s^2}{n'n}$$

To account for the sampling fraction, the goal is to then approximately match this variance $(1-\frac{n}{n})(\frac{s^2}{n})$. It is straightforward to verify the two variances are approximately equal when

$$n' = \frac{n-1}{1-\frac{n}{N}}$$

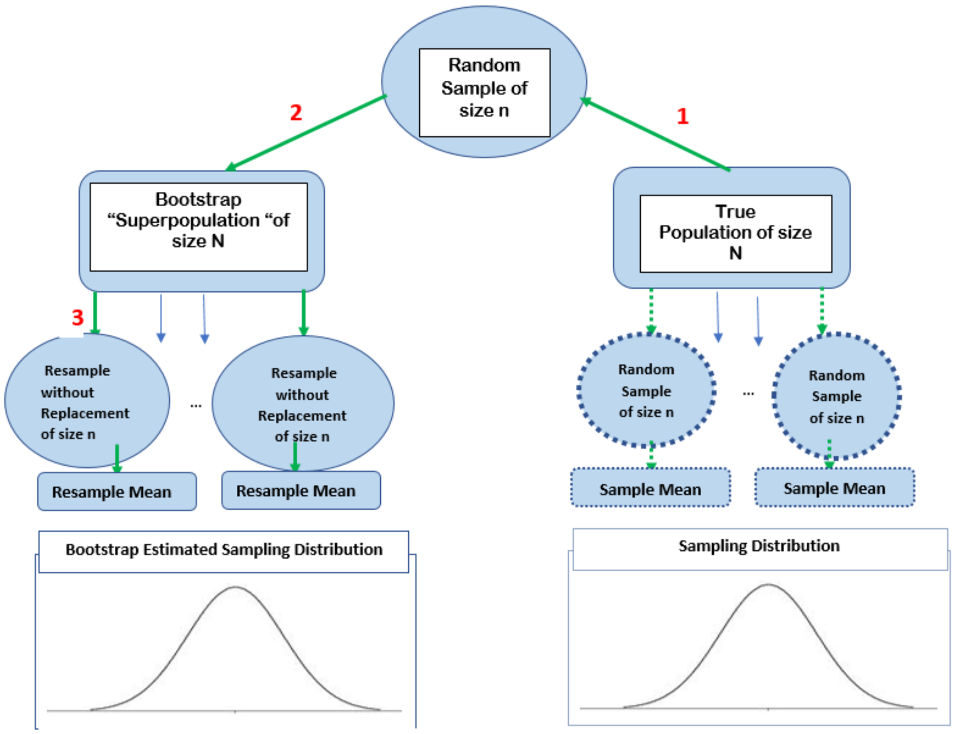
. Hence taking bootstrap resamples with replacement of size n' approximately captures the effect of the dependency among sample values.



The steps for this procedure are the same as the ordinary bootstrap method described earlier except the resample size is adjusted to be n' instead of the original sample size.

4.2 Population and Superpopulation Bootstrap

Our random sample is our best guess of what the population looks like except the size is smaller than the population size. The idea behind this method is to form a "superpopulation" the same size as the population size from our sample and mimic how the original data were sampled from the population.



The steps for this procedure are illustrated above.

1. A random sample of size n is taken from a population of size N.
2. A "superpopulation" of size N is formed by making $\frac{N}{n}$ copies of the sample. If $\frac{N}{n}$ is not an integer then we write $N = kn + l, l < n$, make k copies and add them to l units taken without replacement from the original sample to form a "superpopulation" of size N.
3. The next step is to take B bootstrap without replacement resamples of size n from the "superpopulation".
4. The bootstrap sample mean is calculated from each of the B resamples, and the B bootstrap sample means are used to generate an empirical bootstrap distribution.The percentile method can then be used to genearate approximate confidence intervals.

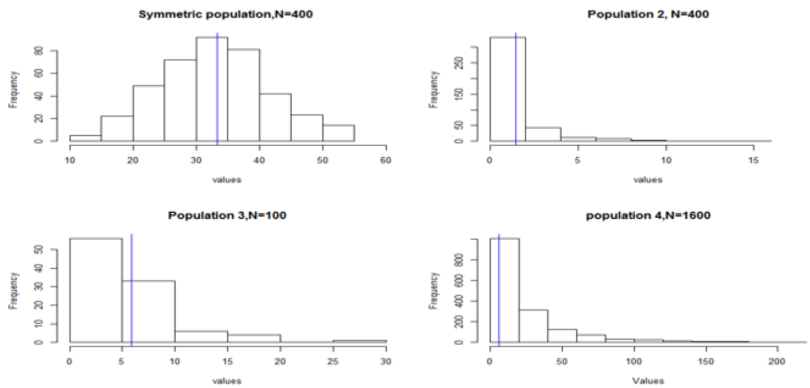
Davison and Hinkley(1997) wrote that the variance of the sample mean under this method is

$$\frac{N(n-1)}{(N-1)n * (1 - \frac{n}{N})s^2}$$

The first factor $\frac{N(n-1)}{(N-1)n}$ is usually close to 1, thereby approximately accounting for the finite population fraction.

5 Simulation Study

Using a simulation study, we compared the coverage probability and precision among the four described methods on four different populations; one symmetric populations of size 400, and three skewed populations of size 1600, 400 and 100.



Blue line: Population mean

The coverage probabilities and average confidence interval widths for approximate 99%, 95% and 90% confidence intervals were computed using the four methods for three different sampling fractions corresponding to sample size equal to 5%, 10% and 15% of the population size N. For a given confidence level, sample size and method, 10,000 simple random samples were taken and 5,000 bootstrap resamples were generated from each sample for the three bootstrap methods. The coverage rate and the average width were then computed from the 10,000 confidence intervals.

6 Results and Discussion

6.1 Results: Coverage Rate and Mean Width

The coverage rate was computed as the percentage of the 10,000 confidence intervals that contained the population mean. For each interval, the difference between the upper bound and lower bound or interval width was recorded. The mean width was calculated as the average of the 10,000 widths.

Symmetric Population(N=400)

	Symmetric Population (N=400)		
	Coverage rates for 99% C.I		
		Sample size	
Method	**20(5% of N)**	**40(10% of N)**	**60 (15% of N)**
t-based	99.07% /(98.88%, 99.26%)	99.14%/(98.96%,99.32%)	99.56%/(99.43%, 99.69%)
Ordinary Bootstrap	98.11%/(97.84%,98.38%)	98.71%/(98.49%,98.93%)	99.4%/(99.25%, 99.55%)
Population Bootstrap	97.91%/(97.63%, 98.19%)	98.37%/(98.12%, 98.62%)	98.88%/(98.67%, 99.09%)
Modified Sample Size	98.07%/(97.80%, 98.34%)	98.51%/(98.27%, 98.75%)	99.00%/(98.80%, 99.20%)
	Coverage rates for 95% C.I		
		Sample size	
Method	**20(5% of N)**	**40(10% of N)**	**60 (15% of N)**
t-based	95.67%/(95.27%, 96.07)	96.1%/(95.72%, 96.48%)	96.97%/(96.63%, 97.31%)
Ordinary Bootstrap	93.88%/(93.41%, 94.35%)	95.16%/(94.74%, 95.58%)	96.45%/(96.09%, 96.81%)
Population Bootstrap	93.20%/(92.71%, 93.69%)	93.95%/(93.48%, 94.42%)	94.76%/(94.32%, 95.20%)
Modified Sample Size	93.82%/(93.35%, 94.29%)	94.28%/(93.82%, 94.74%)	95.15%/(94.73%, 95.57%)
	Coverage ratess for 90% C.I		
		Sample size	
Method	**20(5% of N)**	**40(10% of N)**	**60 (15% of N)**
t-based	91.00%/(90.44%, 91.56%)	91.54%/(90.99%, 92.09%)	92.90%/(92.40%, 93.40%)
Ordinary Bootstrap	88.56%/(87.94%, 89.18%)	90.50%/(89.93%, 91.07%)	92.25%/(91.73%, 92.77%)
Population Bootstrap	87.76%/(87.12%, 88.40%)	88.61%/(87.99%, 89.23%)	89.63%/(89.03%, 90.23%)
Modified Sample Size	88.57%/(87.94%, 89.48%)	89.37%/(88.67%, 89.89%)	90.33%/(89.75%, 90.91%)

Table 1: Coverage rates for the symmetric population

Table 1 shows the coverage rates along with associated 95% confidence intervals for the symmetric population of size 400.The green highlight indicates that the coverage rate is within 1% of the nominal confidence level. The coverage rates by the t-based approach approximately attained the nominal 99% confidence level for all three sample sizes. Those of the ordinary bootstrap and Modified sample size methods were also close the nominal 99% confidence level for all sample sizes. The coverage rates by the population bootstrap approach approximately attained the 99% level of confidence when the sampling fractions were 10% and 15% of the population size. For 95% level of confidence, the coverage rate was higher than the nominal level at the 15% sampling fraction but did approximately attain the desired level of confidence at 10% and 5% sampling fractions. The Modified sample size approach did approximately match the 95% and 90% levels of confidence at 10% and 15% sampling fractions whereas the Population bootstrap method did approximately attain these nominal confidence levels when the sampling fraction was 15%. The coverage rates by the ordinary bootstrap method were close to the 95% and 90% levels of confidence at 10% sampling fraction.

Population 2 (N=400)

	Population 2(N=400)		
	Coverage rates for 99% C.I		
		Sample size	
Method	**20(5% of N)**	**40(10% of N)**	**60 (15% of N)**
t-based	95.72%/(95.32%, 96.12%)	97.25%/96.92%, 97.57%)	98.71%/(98.49%, 98.93%)
Ordinary Bootstrap	94.24%/(93.78%, 94.70%)	97.12%/(96.79%, 97.45%)	98.15%/(97.89%, 98.41%)
population Bootstrap	93.76%/(93.29%, 94.23%)	96.44%/(96.08%, 96.80%)	97.32%/(97.00%, 97.64%)
Modified sample size	94.25%/(93.79%, 94.71%)	96.79%/(96.44%, 97.14%)	97.67%/(97.37%, 97.67%)
	Coverage rates for 95% C.I		
		Sample size	
Method	**20(5% of N)**	**40(10% of N)**	**60 (15% of N)**
t-based	90.8%/(90.23%, 91.37%)	93.2%/(92.71%, 93.89%)	94.55%/(94.10%, 94.99%)
Ordinary Bootstrap	88.49%/(87.86%, 89.12%)	92.59%/(92.08%, 93.10%)	94.37%/(93.92%, 94.82%)
population Bootstrap	87.6%/(86.95%, 88.25%)	91.27%/(90,72%, 91.82%)	92.66%/(92.15%, 93.17%)
Modified sample size	88.42%/(87.79%, 89.05%)	91.91%/(91.38%, 92.44%)	92.98%/(92.48%, 93.48%)
	Coverage ratess for 90% C.I		
		Sample size	
Method	**20(5% of N)**	**40(10% of N)**	**60 (15% of N)**
t-based	85.6%/(84.91%, 86.29%)	88.95%/(88.33%, 89.56%)	90.65%/(90.08%, 91.22%)
Ordinary Bootstrap	82.88%/(82.14%, 83.62%)	87.88%/(87.24%, 88.52%)	90.24%/(89.66%, 90.82%)
population Bootstrap	82.05%/(81.30%, 82.80%)	86.32%/(85.64%, 86.99%)	87.58%/(86.34%, 88.22%)
Modified sample size	82.84%/(82.10%, 83.58%)	87.00%/(86.34%, 87.66%)	88.1%/(87.46%, 88.73%)

Table 2: Coverage rates for right-skewed population (400)

For this population, only the t-based method and the ordinary bootstrap approach approximately attained the nominal confidence levels when the sampling fraction was 15% of the population size.

Population 3 (N=100)

	Population 3(N=100)		
	Coverage rates for 99% C.I		
		Sample size	
Method	**5(5% of N)**	**10(10% of N)**	**15 (15% of N)**
t-based	98.27%/(98.01%, 98.53%)	97.85%/(97.57%, 98.13%	97.88%/(97.60%, 98.16%)
Ordinary Bootstrap	86.4%/(85.73%, 87.07%)	94.33%/(93.88%, 94.78%	96.2%/(95.83%, 96.57%)
Population Bootstrap	86.06%/(85.38%, 87.07%)	93.5%/(93.02%, 93.98%)	94.85%/(94.41%, 95.28%)
Modified sample size	87.03%/(86.37&, 87.69%)	94.34%/(93.89%, 94.79%	95.59%/(95.19%, 95.99%)
	Coverage rates for 95% C.I		
		Sample size	
Method	**5(5% of N)**	**10(10% of N)**	**15 (15% of N)**
t-based	92.32%/(91.80%, 92.84%)	93.06%/(92.56%, 93.56%	93.64%/(93.16%, 94.11%)
Ordinary Bootstrap	80.31%/(79.53%, 81.09%)	88.37%/(87.74%, 89.01%	91.07%/(90.51%, 91.62%)
Population Bootstrap	79.93%/(79.14%, 80.71%)	86.96%/(86.30%, 87.62%	89.12%/(88.51%, 89.73%)
Modified sample size	83.69%/(82.96%, 84.41%)	88.48%/(87.85%, 89.11%	90.15%/(89.57%, 90.73%)
	Coverage rates for 90% C.I		
		Sample size	
Method	**5(5% of N)**	**10(10% of N)**	**15 (15% of N)**
t-based	87.19%/(86.53%, 87.85%)	88.18%/(87.55%, 88.81%	89.33%/(88.72%, 89.94%)
Ordinary Bootstrap	75.75%/(74.91%, 76.59%)	83.05%/(82.31%, 83.78%	86.31%/(85.63%, 86.98%)
Population Bootstrap	75.22%/(74.37%, 76.07%)	81.27%/(80.51%, 82.03%	83.69%/(82.97%, 84.41%)
Modified sample size	79.14%/(78.34%, 79.93%)	83.1%/(82.37%, 83.83%)	85.12%/(84.42%, 85.82%)

Table 3: Coverage rates for right-skewed population (100)

From Table 3, only coverage rates by the t-based method were within 1% of the 99% and 90% levels of confidence at 5% and 15% sampling fractions respectively.

Population 4 (N=1600)

	Population 4(N=1600)		
	Coverage rates for 99% C.I		
		Sample size	
Method	**80(5% of N)**	**160(10% of N)**	**240 (15% of N)**
t-based	98.05%/(97.78%, 98.32%)	98.68%/(98.46%, 98.90%)	99.05%/(98.86%, 99.24%)
Ordinary Bootstrap	98.28%/(98.03%, 98.53%)	98.89%/(98.68%, 99.09%)	99.15%/(98.97%, 99.33%)
Population Bootstrap	98.05%/(97.78%, 98.32%)	98.3%/(98.05%, 98.55%)	98.53%/(98.29%, 98.77%)
Modified sample size	98.14%/(97.88%, 98.40%)	98.46%/(98.22%, 98.70%)	98.57%/(98.34%, 98.80%)
	Coverage rates for 95% C.I		
		Sample size	
Method	**80(5% of N)**	**160(10% of N)**	**240(15% of N)**
t-based	94.39%/(93.94%, 94.84%)	95.08%/(94.66%, 95.50%)	96.01%/(95.63%, 96.39%)
Ordinary Bootstrap	94.31%/(93.86%, 94.76%)	95.1%/(94.68%, 95.52%)	96.01%/(95.63%, 96.39%)
Population Bootstrap	93.73%/(93.25%,94.21%)	93.85%/(93.38%, 94.32%)	94.33%/(93.88%, 94.78%)
Modified sample size	93.93%/(93.25%, 94.21%)	93.98%/(93.51%, 94.44%)	94.49%/(94.04%, 94.94%)
	Coverage rates for 90% C.Is		
		Sample size	
Method	**80(5% of N)**	**160(10%of N)**	**240 (15% of N)**
t-based	90.02%/(89.43%, 90.61%)	90.5%/(89.93%, 91.07%)	91.93%/(91.40%, 92.46%)
Ordinary Bootstrap	89.54%/(88.94%, 90.13%)	90.31%/(89.73%, 90.89%)	91.93%/(91.40%, 92.46%)
Population Bootstrap	88.8%/(88.18%, 89.42%)	88.63%/(88.01%, 89.25%)	89.35%.(88.75%, 89.95%)
Modified sample size	89.04%/(88.43%, 89.65%)	88.67%/(88.05%, 89.29%)	89.48%/(88.88%, 90.08%)

Table 4: Coverage rates for right-skewed population (1600)

The coverage rates by the four methods approximately attained the 99% nominal level of confidence for all sample sizes. The t-based and the ordinary bootstrap methods were close to the three nominal levels of confidence for all three sample sizes. The coverage rates by the two finite population correction methods were however closed to the 95% and 90% levels of confidence at a sampling fraction of 15% of the population size.

Across the four populations, the t-based method has the highest coverage for a given sample size and confidence level followed by the ordinary bootstrap, with the population bootstrap method has the lowest coverage rate among the four methods. The pattern among the four methods does not seem to depend on confidence level as the order generally stayed the same across the three confidence levels. Predictably, for a given confidence level and method, the coverage rate increased as the sampling fraction increases.

	Symmetric Population (N=400)			Population 2(N=400)		
	Average widths/SE for 99% C.I			Average widths/SE for 99% C.I		
		Sample size			Sample size	
Method	20(5% of N)	40(10% of N)	60 (15% of N)	20(5% of N)	40(10% of N)	60 (15% of N)
t-based	11.03/1.58	7.41/0.71	5.96/0.45	2.35/0.92	1.61/0.46	1.32/0.30
Ordinary Bootstrap	9.60/1.37	6.92/0.67	5.69/0.44	2.01/0.77	1.50/0.42	1.25/0.28
Population Bootstrap	9.34/1.34	6.57/0.64	5.25/0.41	1.96/0.75	1.42/0.39	1.15/0.26
Modified Sample Size	9.60/1.37	6.69/0.65	5.31/0.41	2.01/0.77	1.45/0.41	1.17/0.26
	Average widths/SE for 95% C.I			Average widths/SE for 95% C.I		
		Sample size			Sample size	
Method	20(5% of N)	40(10% of N)	60 (15% of N)	20(5% of N)	40(10% of N)	60 (15% of N)
t-based	8.07/1.15	5.53/0.53	4.48/0.34	1.72/0.68	1.21/0.34	0.99/0.22
Ordinary Bootstrap	7.35/1.05	5.29/0.51	4.35/0.33	1.55/0.60	1.15/0.32	0.96/0.22
Population Bootstrap	7.17/1.03	5.02/0.49	4.01/0.31	1.51/0.58	1.09/0.31	0.88/0.20
Modified Sample Size	7.35/1.05	5.10/0.49	4.06/0.31	1.55/0.60	1.11/0.31	0.89/0.20
	Average widths/SE for 90% C.I			Average widths/SE for 90% C.I		
		Sample size			Sample size	
Method	20(5% of N)	40(10% of N)	60 (15% of N)	20(5% of N)	40(10% of N)	60 (15% of N)
t-based	6.67/0.95	4.61/0.44	3.74/0.28	1.42/0.56	1.01/0.28	0.83/0.18
Ordinary Bootstrap	6.18/0.89	4.44/0.43	3.65/0.28	1.31/0.51	0.97/0.27	0.80/0.18
Population Bootstrap	6.03/0.87	4.22/0.41	3.37/0.26	1.28/0.50	0.92/0.26	0.74/0.17
Modified Sample Size	6.18/0.89	4.28/0.42	3.41/0.26	1.31/0.51	0.93/0.26	0.75/0.17

	Population 3(N=100)			Population 4(N=1600)		
	Average widths/SE for 99% C.I			Average widths/SE for 99% C.I		
		Sample size			Sample size	
Method	5(5% of N)	10(10% of N)	15 (15% of N)	80(5% of N)	160(10% of N)	240 (15% of N)
t-based	16.31/8.66	8.63/3.22	6.59/1.97	16.74/2.81	11.76/1.37	9.59/0.88
Ordinary Bootstrap	7.64/4.01	6.29/2.30	5.41/1.60	16.10/2.71	11.52/1.36	9.46/0.88
Population Bootstrap	7.40/3.84	5.98/2.18	4.97/1.46	15.69/2.64	10.92/1.29	8.71/0.82
Modified sample size	8.22/4.21	6.30/2.30	5.24/1.55	15.82/2.65	10.96/1.29	8.74/0.81
	Average widths/SE for 95% C.I			Average widths/SE for 95% C.I		
		Sample size			Sample size	
Method	5(5% of N)	10(10% of N)	15 (15% of N)	80(5% of N)	160(10% of N)	240(15% of N)
t-based	9.83/5.23	6.01/2.24	4.75/1.42	12.62/2.11	8.91/1.04	7.29/0.67
Ordinary Bootstrap	6.05/3.16	4.87/1.79	4.16/1.23	12.31/2.07	8.79/1.03	7.21/0.67
Population Bootstrap	5.97/3.13	4.64/1.71	3.85/1.15	12.00/2.01	8.34/0.98	6.65/0.62
Modified sample size	6.70/3.57	4.87/1.79	4.03/1.20	12.09/2.03	8.36/0.98	6.67/0.62
	Average widths/SE for 90% C.I			Average widths/SE for 90% C.I		
		Sample size			Sample size	
Method	5(5% of N)	10(10% of N)	15 (15% of N)	80(5% of N)	160(10%of N)	240 (15% of N)
t-based	7.55/4.01	4.87/1.82	3.90/1.17	10.55/1.77	7.46/0.87	6.10/0.56
Ordinary Bootstrap	5.26/2.82	4.13/1.53	3.50/1.05	10.34/1.74	7.38/0.87	6.06/0.56
Population Bootstrap	5.16/2.75	3.94/1.47	3.25/0.97	10.08/1.69	7.00/0.82	5.59/0.52
Modified sample size	5.70/2.90	4.13/1.54	3.39/1.01	10.15/1.70	7.02/0.82	5.60/0.52

Table 5: Average widths and their standard errors across the 4 populations

The order is reversed when it comes to the mean widths for the four methods. The population bootstrap method has the smallest mean width for a given sample size and confidence level followed closely by the modified resample size method. For any method, however, the mean width decreases as the sampling fraction was increased.

6.2 Discussion

In constructing confidence intervals, the goal of every researcher is to first and foremost match the desired level of confidence at least approximately. The results showed that even though the two finite population correction methods generate narrower and more precise confidence intervals than the t-based and ordinary bootstrap approaches, in most instances it was observed that such confidence intervals might be too narrow to achieve the desired coverage level. On the other hand, it was also observed that the t-based and the ordinary bootstrap methods may generate confidence intervals that might be too wide to attain the desired level of confidence.

The methods especially the finite population correction methods seem to work better in some populations than others. Generally, the coverage probabilities from the symmetric population were higher were higher than the three skewed populations. The fourth population of size 1600 had higher coverage probabilities than the other two skewed populations. This is indication that population characteristics like skewness may play role in how well these methods work and when they should be applied. Most of the coverage rates by the four methods did not achieved the desired levels of confidence in population 3. Therefore, for some skewed population distributions, both the traditional methods and the finite population correction method may not achieve the desire levels of confidence. In such situations, inference about other measures of center like the population median might be explored.

Predictably, it was revealed that sampling fraction plays a role in the utility of these methods. The coverage rates seem to increase as the sampling fraction was increased across the four populations whereas the mean widths decreased as the sampling fraction was increased. As sampling fraction increases, we get more information about the population that resulted in having a better coverage rates and more precise confidence intervals.

Looking at the results, it did not appear the choice of confidence level had any effect on coverage probability and precision of a particular method as there was no detectable pattern across the three different levels of confidence used in the study. Conducting a formal test to look these relationships might be more appropriate in future studies.

	Sample size			Sample size			Sample size	
Method	**40(10% of N)**	**60 (15% of N)**		**40(10% of N)**	**60 (15% of N)**		**40(10% of**	**60 (15% of N)**
t-based	99.14%/7.40	99.56%/5.96		96.1%/5.53	96.97%/4.48		91.54%/4.	92.9%/3.74
Ordinary Bootstrap	98.71%/6.92	99.4%/5.70		95.16%/5.29	96.45%/4.35		90.5%/4.4	92.25%/3.65
Population Bootstrap	98.37%/6.57	98.88%/5.25		93.95%/5.02	94.76%/4.01		88.61%/4.	89.63%/3.37
Modified sample size	98.51%/6.68	99%/5.31		94.28%/5.10	95.15%/4.06		89.37%/4.	90.33%/3.41

Another look at Table 1 indicates both the four methods had comparable coverage rates for sampling fractions of 10% and 15% except in some cases the t-based and the ordinary bootstrap methods generated confidence intervals that were too wide to achieve the desired levels of confidence. It suggests that if an underlying population distribution is assumed to be symmetric, then employing the two described population correction methods may yield more precise approximate confidence intervals than the t-based and ordinary bootstrap methods when the sampling fraction is at least 10% of the population size.

	Population 4(N=1600)				
	Sample size			Sample size	
Method	160(10% of N	240(15% of N)		160(10%o	240 (15% of N)
t-based	95.08%/8.91	96.01%/7.27		90.5%/7.4	91.93%/6.10
Ordinary Bootstrap	95.1%8.79	96.01%/7.21		90.31%/7.	91.93%/6.06
Population Bootstrap	93.85%/8.34	94.33%/6.65		88.63%/7.	89.35%.5.59
Modified sample size	93.98%/8.35	94.49%/6.67		88.67%/7.	89.48%/5.60

For some skewed populations, similar to the population 4, the two described population correction methods may yield more precise approximate confidence intervals than the t-based and ordinary bootstrap methods when the sampling fraction is at least 15% of the population. For a smaller sampling fraction of 5% or less, the finite population methods may generate too narrow confidence intervals achieve the approximate confidence levels. In such situations, the t-based and the ordinary bootstrap methods might be more appropriate.

7 Improvement and Future Work

Future work could look at conducting formal tests such as regression analysis to investigate the effect of the four methods and levels of confidence on coverage rate and average width. Studies focusing on more populaions could help elucidate the effect of population on these methods and these studies could also look at the threshold sampling fractions to achieve 100% coverage rate.

8 References

1. Efron, B. and Tibshirani, R. (1986) Bootstrap methods for standard errors, confidence intervals, and other measures of statistical accuracy. Statistical Science 1(1): 54 - 77.

2. DiCiccio, T. and Tibshirani, R. (1987) Bootstrap confidence intervals and bootstrap approximations. Journal of the American Statistical Association 82 (397): 163 - 170 .

3. Efron, B. (1987) Better bootstrap confidence intervals. Journal of the American Statistical Association 82 (397): 171 - 200.

4. Efron, B. (1988) Bootstrap confidence intervals: Good or Bad? Psychological Bulletin 104(2): 293 - 296.

5. Hinkley, D. V. (1988) Bootstrap methods. J. R. Statist. Soc. B 50(3): 321 - 337

6. Davison, A.C. and Hinkley, D. V. (1997) Bootstrap methods and their application. Cambridge University Press. pp. 92 - 100