A better version of review problem

(4) For $E = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, invert $A = \left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ \hline 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \end{array}\right]$

by using 2x2 block version of Gauss-Jacobi algorithm.

<u>Solution</u>: $[A \mid I] = \left[\begin{array}{c|c|c|c} E & 0 & I & 0 \\ \hline 2I & I & 0 & I \end{array}\right]$ 1st (block) pivot

will need $E^{-1} = \frac{1}{1}\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

Gauss step 1 $\rightsquigarrow$ $(2) - 2E^{-1}(1)$ $\left[\begin{array}{c|c|c|c} E & 0 & I & 0 \\ 0 & I & -2E^{-1} & I \end{array}\right]$

Gauss Step 2 $\rightsquigarrow$ $E^{-1}\cdot(1)$ $\left[\begin{array}{cc|cc} I & 0 & E^{-1} & 0 \\ 0 & I & -2E^{-1} & I \end{array}\right]$ Thus $A^{-1} = \begin{bmatrix} E^{-1} & 0 \\ -2E^{-1} & I \end{bmatrix}$

(<u>NOTE</u>: No Jacobi steps needed.)

<u>CHECK</u>: (by block multiplication) $AA^{-1} = \begin{bmatrix} E & 0 \\ 2I & I \end{bmatrix}\begin{bmatrix} E^{-1} & 0 \\ -2E^{-1} & I \end{bmatrix} = \begin{bmatrix} EE^{-1} & 0 \\ \underbrace{2IE^{-1} - I2E^{-1}}_{0} & I \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$

$A^{-1}A = \text{- - - - - -} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$

Final answer $A^{-1} = \left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ \hline -2 & 0 & 1 & 0 \\ 4 & -2 & 0 & 1 \end{array}\right]$