0	1	2	3	4	5	6	7	8	
10	15	20	15	15	20	15	15	15	

Math 221 Final (6 May 2010)

NAME:

Show all work (unless instructed otherwise). Good Luck!

0.[10pts] Indicate whether the statements are true or false.

(T F) Eigenvectors with different eigenvalues are independent.

(T F) $\det(AB) = \det(A)\det(B)$.

(T F) A 5×3 matrix cannot be of rank 4.

(T F) If $Ax = b$ can be solved for some b then it can be solved for all b.

(T F) $(AB)^T = A^T B^T$.

1.[15pts] Let V be the subspace spanned by $v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $v_2 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$, $v_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$.

a) For what a does $(a, a, 1)$ belong to V?

b) Find a basis of V.

c) The subspace $V^{\perp}$ orthogonal to V is the null space of some matrix A. Write out A. (Do not find $N(A)$.)

$A =$

2.[20pts] For

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

a) Use Gauss-Jordan Elimination to find A^{-1}. (Indicate the row operations used.)

b) Write out the LU-decomposition of A (based on a)).

c) Use A^{-1} to solve $Ax = b$ when $b = (1, 2, 3)$. (Use the suggested method.)

3.[15pts] For $A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$

a) Use the Laplace expansion about the first row to compute $\det(A)$.

b) Test ~~if A~~ the following matrix is positive definite: (Goofed A is not symmetric!)

$$\begin{bmatrix} 2 & 2 & 0 \\ 2 & 4 & -2 \\ 0 & -2 & 4 \end{bmatrix}$$

c) $(-1, 1, 0)$ is an eigenvector of A. What is its eigenvalue?

4.[15pts] The diagonalization of some B is given by

$$B = \begin{pmatrix} 0 & 0 & -1 \\ 2 & -1 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ -1/3 & -1/3 & 2/3 \\ -1 & 0 & 0 \end{pmatrix}$$

a) The eigenvalues of B are [← *Fill the blank.*]

b) The eigenvectors of B are [← *Fill the blank.*]

c) Based on b), decide if B is symmetric (without computing B). Explain.

d) Complete the three missing diagonal entries in

$$B^3 = \begin{pmatrix} 0 & 0 & -1 \\ 2 & -1 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} \; & 0 & 0 \\ 0 & \; & 0 \\ 0 & 0 & \; \end{pmatrix} \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ -1/3 & -1/3 & 2/3 \\ -1 & 0 & 0 \end{pmatrix}$$

5.[20pts] Consider the matrix

$$A = \begin{bmatrix} 3 & 0 & 4 \\ 0 & 2 & 0 \end{bmatrix}$$

a) Compute the singular values of A (by using AA^T).

b) A maps the unit ball to an ellipse with semi-axes ______ , ______ [← *Fill the blanks.*]

c) Find the unique x_r in the row space solving $Ax_r = b$ when $b = (7, 2)$.

d) Give the complete solution to $Ax = b$ when $b = (7, 2)$.

6.[15pts] Consider the system $\begin{bmatrix} 1 & 2 \\ 1 & 0 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

a) What equation on b_1, b_2, b_3 assures solvability of the system?

b) Is there b for which the system has infinitely many solutions? Explain.

c) Give the least squares solution when $b = (0, 6, 6)$.

7.[15pts] Find the formula for a_n given by $a_0 = 0$, $a_1 = 1$ and

$$a_{n+2} = 3a_{n+1} + 4a_n \qquad \text{for } n = 0, 1, 2, 3, \ldots.$$

8.[15pts] Consider

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

a) find an orthonormal basis of the column space $C(A)$ of A.

b) Use a) to find the projection of $(1, 1, 1)$ onto A.

c) Find the QR decomposition of A.