0	1	2	3	4	5	

Math 221 Second Exam (31 March 2010)

Show all work (unless instructed otherwise). NAME:
Good Luck!

0. Circle **True** or **False** without explanation:

(**T** or **F**) If vectors u and v are in a subspace then so is $u + 2v$.

(**T** or **F**) There is a 2×2 matrix A with $N(A) = C(A)$.

(**T** or **F**) Two planes can be orthogonal in $\mathbf{R}^4$, but not in $\mathbf{R}^3$.

(**T** or **F**) If $2v_1 + 3v_2 + 4v_3 = 0$ then the vectors v_1, v_2, v_3 are dependent.

(**T** or **F**) If A is 4×3 and B is 3×4, then $C = AB$ cannot be invertible.

1. The matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$$

can be written as uv^T for some vectors u, v.

a) Complete: The rank of A must be ....

b) Find the vectors u, v.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix} = \overset{u}{\begin{bmatrix} \ldots \\ \ldots \\ \ldots \end{bmatrix}} \overset{v^T}{[\ldots, \quad \ldots, \quad \ldots]}$$

2. Find the dimension and a basis of the subspace that consists of all 2×2 matrices of the form $A = \begin{bmatrix} a & a \\ b & c \end{bmatrix}$ (a, b, c are scalars.)

3. You are given A and its row reduced form R:

$$A = \begin{bmatrix} 1 & 2 & 1 & 5 \\ 1 & -2 & -1 & -1 \\ 2 & -2 & -1 & 1 \end{bmatrix} \qquad R = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 2 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

a) Complete: The rank of A is ...

b) Complete: the column space $C(A)$ has dimension ... and a basis ...

list basis vectors here

c) Complete: the row space $C(A^T)$ has dimension ... and a basis ...

d) Compute a basis for the null space $N(A)$.
Show Work Here:

e) Complete: The dimension of $N(A^T)$ is .... and its basis is $(1, 3, \ldots)$
Show Work:

fill in.

f) State the condition on $b = (b_1, b_2, b_3)$ guaranteeing that $Ax = b$ has a solution. (Use e).)

4. Consider $u = (1,1,1)$, $v = (1,1,0)$, and $b = (1,0,1)$.
a) Verify by a computation that b is not in the plane spanned by u, v.

b) Find the QR-decomposition of the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$ with columns u, v.

c) Compute the projection of b onto the plane of u, v. (Use the columns of Q.)

d) Find the least squares solution to $Ax = b$.

5. For

$$A = \begin{bmatrix} 1 & -2 \\ 2 & -4 \\ 3 & -6 \end{bmatrix}$$

a) Find the x_r in the row space $C(A^T)$ solving $Ax_r = b$ for $b = (9, 18, 27)$.

b) Find the complete solution to $Ax = b$.