Review Problems

① Consider $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 2 & -9 \end{bmatrix}$

a) Find the LU and LDU factorizations of A

b) Use the LU factorization to solve $Ax = \begin{bmatrix} 5 \\ 14 \\ 21 \end{bmatrix}$

c) Find A^{-1} and solve the same system using A^{-1}!
by using Gauss-Jordan el.

② In ①, replace the 3 on the diagonal by a parameter a, that is let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & a & 2 \\ 3 & 2 & -9 \end{bmatrix}$.

a) For what value of a, Gauss Elimination requires row switching?

b) Carry out the $PA = LDU$ factorization. (Find P, L, U.)

c) A is symmetric. Is it true that $U = L^T$. Why?

③ Write a permutation matrix P acting as follows:
$1 \mapsto 4,\ 2 \mapsto 1,\ 3 \mapsto 3,\ 4 \mapsto 2.$
and write out (without eliminating) P^{-1}!

④ For $E = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ find E^{-1} and then invert $A = \left[\begin{array}{cc|cc} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ \hline 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{array}\right]$
by using 2×2 inverse formula and the block decomposition.

⑤ Simplify: a) $B\left((AB)^{-1}A + I\right) - B$ b) $A \cdot (B^{-1})^T (A+B)^T (A^{-1})^T$

(6) For $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 4 \\ 1 & 0 & 6 \end{bmatrix}$ compute

a) column 2 of AB

b) row 2 of AB (without computing whole matrices!)

c) row 2 of A^2 and row 2 of A^3.

(7) Multiply the A, B from (6) "Columns by rows". (also solve #26 p79)

(8) a) Find the pivots for the system
$$\begin{cases} x + y + z + t = b_1 \\ z + t = b_2 \\ z + t = b_3 \end{cases}.$$

b) Find $\vec{b}$ for which it has no-solutions.

c) Find $\vec{b}$ for which it has ∞-many solutions.

d) Draw the column picture for $b = \begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix}$.

e) What would the row picture consist of?

(9) Illustrate with a nice sketch the addition: $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

(10) For what value of the parameter μ are $\begin{pmatrix} \mu \\ 2 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} \mu \\ \mu \\ 2 \end{pmatrix}$ perpendicular?

(11) Give examples of triples of vectors $\vec{u}, \vec{v}, \vec{w}$ in 3D such that their linear combinations form: a point, a line, a plane, all of 3D space.
Which is the case for $\vec{u} = \begin{pmatrix} 2 \\ 1 \\ 7 \end{pmatrix}$, $\vec{v} = \begin{pmatrix} 4 \\ 5 \\ -1 \end{pmatrix}$, $\vec{w} = \begin{pmatrix} 0 \\ -3 \\ 15 \end{pmatrix}$?

(12) For depicted $\vec{u}, \vec{v}$, shade the region filled by the linear combinations $c\vec{u} + d \cdot \vec{v}$ with $c, d \geq 0$.