

5.5 Replicated Latin Square Designs

- The experimenter is concerned with a single factor having p levels where p is small (e.g., $p = 2, 3, 4$).
- For each of these cases the disadvantage of using a latin square design (LSD) is the small number of degrees of freedom for error (df_E). For $p = 2, 3, 4$, the df_E are 0, 2, and 4 for the 2×2 , 3×3 , and 4×4 latin square designs, respectively.
- Thus when p is small, it is desirable to replicate a $p \times p$ latin square to increase the df_E .
- We will study three forms of a **replicated latin squares design (RLSD)** which are based on whether or not the researcher can use the same row and column blocks across the replicates. For each replicate a random LSD is selected using the process described earlier in the course.
- Suppose a RLSD having n replicates is run. In each of the associated models μ is the baseline mean and y_{ijkl} and ϵ_{ijkl} are the response and random error associated with row i , treatment j , column k in replicate l . We assume $\epsilon_{ijkl} \sim \text{IID } N(0, \sigma^2)$.
- We are also assuming that there is no interaction among treatments and replicates. That is, we have an *additive model*.
- Suppose an engineer wants to compare the mean viscosity of 4 different resin compounds. Each compound contains an inert liquefying ingredient (ILI) and a technician must be involved in the resin extruding process. The following designs contain 3 “replicates” defined as follows.
 - **RLSD-1 Design:** 4 random batches of ILI and 4 technicians are selected. A latin square design is run for each replicate. The same 4 batches of ILI and the same 4 technicians are used in each of the 3 replicates.
 - **RLSD-2 Design:** 12 random batches of ILI and 4 technicians are selected. A latin square design is run for each replicate with 4 different batches of ILI used in each replicate. However, the same 4 technicians are used in each of the 3 replicates.
 - **RLSD-3 Design:** 12 random batches of ILI and 12 technicians are selected. A latin square design is run for each replicate. Four different batches of ILI and a 4 different technicians are used in each of the 3 replicates.

RLSD-1: The same row and column blocks appear in each replicate. The model is:

$$y_{ijkl} = \quad (38)$$

where ρ_l is the l^{th} replicate effect, α_i is the i^{th} row block effect, β_k is the k^{th} column block effect, and τ_j is the j^{th} treatment effect.

Analysis of Variance for a Replicated Latin Square, Case 1

| Source of Variation | Sum of Squares | Degrees of Freedom | Mean Square | F_0 |
|---------------------|--|-------------------------|--|--------------------------------------|
| Treatments | $\frac{1}{np} \sum_{j=1}^p y_{j..}^2 - \frac{y_{...}^2}{N}$ | $p - 1$ | $\frac{SS_{\text{Treatments}}}{p - 1}$ | $\frac{MS_{\text{Treatment}}}{MS_E}$ |
| Rows | $\frac{1}{np} \sum_{i=1}^p y_{i..}^2 - \frac{y_{...}^2}{N}$ | $p - 1$ | $\frac{SS_{\text{Rows}}}{p - 1}$ | |
| Columns | $\frac{1}{np} \sum_{k=1}^p y_{.k.}^2 - \frac{y_{...}^2}{N}$ | $p - 1$ | $\frac{SS_{\text{Columns}}}{p - 1}$ | |
| Replicates | $\frac{1}{p^2} \sum_{l=1}^n y_{.l.}^2 - \frac{y_{...}^2}{N}$ | $n - 1$ | $\frac{SS_{\text{Replicates}}}{n - 1}$ | |
| Error | Subtraction | $(p - 1)[n(p + 1) - 3]$ | $\frac{SS_E}{(p - 1)[n(p + 1) - 3]}$ | |
| Total | $\sum \sum \sum \sum y_{ijkl}^2 - \frac{y_{...}^2}{N}$ | $np^2 - 1$ | | |

RLSD-2: The blocks are identical for one blocking factor but are different for the other blocking factor across the replicates. Without loss of generality, we will assume the column blocks are identical but the row blocks vary across replicates, then the model is:

$$y_{ijkl} = \quad (39)$$

where ρ_l is the l^{th} replicate effect $\alpha_{i(l)}$ is the effect of row block i within replicate l , β_k is the effect of column block k , and τ_j is the j^{th} treatment effect.

Analysis of Variance for a Replicated Latin Square, Case 2

| Source of Variation | Sum of Squares | Degrees of Freedom | Mean Square | F_0 |
|---------------------|--|--------------------|---------------------------------|--------------------------------|
| Treatments | $\frac{1}{np} \sum_{j=1}^p y_{j..}^2 - \frac{y_{...}^2}{N}$ | $p - 1$ | $\frac{SS_{Treatments}}{p - 1}$ | $\frac{MS_{Treatments}}{MS_E}$ |
| Rows | $\frac{1}{p} \sum_{l=1}^n \sum_{i=1}^p y_{i..l}^2 - \sum_{l=1}^n \frac{y_{...l}^2}{p^2}$ | $n(p - 1)$ | $\frac{SS_{Rows}}{n(p - 1)}$ | |
| Columns | $\frac{1}{np} \sum_{k=1}^p y_{..k}^2 - \frac{y_{...}^2}{N}$ | $p - 1$ | $\frac{SS_{Columns}}{p - 1}$ | |
| Replicates | $\frac{1}{p^2} \sum_{l=1}^n y_{...l}^2 - \frac{y_{...}^2}{N}$ | $n - 1$ | $\frac{SS_{Replicates}}{n - 1}$ | |
| Error | Subtraction | $(p - 1)(np - 1)$ | $\frac{SS_E}{(p - 1)(np - 1)}$ | |

RLSD-3: The blocks are different for both blocking factors across the replicates.

$$y_{ijkl} = \quad (40)$$

where ρ_l is the l^{th} replicate effect $\alpha_{i(l)}$ is the effect of row block i within replicate l , $\beta_{k(l)}$ is the effect of column block k within replicate l , and τ_j is the j^{th} treatment effect.

Analysis of Variance for a Replicated Latin Square, Case 3

| Source of Variation | Sum of Squares | Degrees of Freedom | Mean Square | F_0 |
|---------------------|--|-------------------------|--------------------------------------|-------------------------------|
| Treatments | $\frac{1}{np} \sum_{j=1}^p y_{j..}^2 - \frac{y_{...}^2}{N}$ | $p - 1$ | $\frac{SS_{Treatments}}{p - 1}$ | $\frac{MS_{Treatment}}{MS_E}$ |
| Rows | $\frac{1}{p} \sum_{l=1}^n \sum_{i=1}^p y_{i..l}^2 - \sum_{l=1}^n \frac{y_{...l}^2}{p^2}$ | $n(p - 1)$ | $\frac{SS_{Rows}}{n(p - 1)}$ | |
| Columns | $\frac{1}{p} \sum_{l=1}^n \sum_{k=1}^p y_{..kl}^2 - \sum_{l=1}^n \frac{y_{...l}^2}{p^2}$ | $n(p - 1)$ | $\frac{SS_{Columns}}{n(p - 1)}$ | |
| Replicates | $\frac{1}{p^2} \sum_{l=1}^n y_{...l}^2 - \frac{y_{...}^2}{N}$ | $n - 1$ | $\frac{SS_{Replicates}}{n - 1}$ | |
| Error | Subtraction | $(p - 1)[n(p - 1) - 1]$ | $\frac{SS_E}{(p - 1)[n(p - 1) - 1]}$ | |

The following table contains the data for the following RLSD-1 and RLSD-3 examples.

| Operator | Replicate 1 | | | | Replicate 2 | | | |
|----------|-------------|------|------|-----|-------------|------|-----|------|
| | Days | | | | Days | | | |
| | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| | B | C | A | D | D | C | A | B |
| 1 | 810 | 1080 | 700 | 910 | 840 | 1050 | 775 | 805 |
| | C | D | B | A | A | D | B | C |
| 2 | 1100 | 880 | 780 | 600 | 670 | 930 | 720 | 1035 |
| | D | A | C | B | C | B | D | A |
| 3 | 840 | 540 | 1055 | 830 | 980 | 700 | 810 | 610 |
| | A | B | D | C | B | A | C | D |
| 4 | 650 | 740 | 1025 | 900 | 860 | 730 | 970 | 900 |

RLSD-1 Example: A manufacturing firm investigated the breaking strengths of components made from raw materials purchased from 4 suppliers (A, B, C, D). Data was collected from 2 replicates of a 4 × 4 latin square design. The blocking factors were days and operators. The same four operators were used in both replicates. Each replicate was also run on the same four days with replicated values taken during the morning and afternoons of these four days.

SAS code for RLSD-1 Example

```
DM 'LOG; CLEAR; OUT; CLEAR;';
ODS GRAPHICS ON;
* ODS PRINTER PDF file='C:\COURSES\ST541\RLSD1.PDF';
OPTIONS NODATE NONUMBER;

*****;
*** REPLICATED LATIN SQUARE EXAMPLE RLSD-1 ***;
*****;
DATA rlsd1 ;
  DO rep      = 1 to 2;
  DO operator = 1 TO 4;
  DO day      = 1 TO 4;
    INPUT strength supplier $ @@; OUTPUT;
  END; END; END;
CARDS;
810 B 1080 C 700 A 910 D 1100 C 880 D 780 B 600 A
840 D 540 A 1055 C 830 B 650 A 740 B 1025 D 900 C
840 D 1050 C 775 A 805 B 670 A 930 D 720 B 1035 C
980 C 700 B 810 D 610 A 860 B 730 A 970 C 900 D

PROC GLM DATA=rlsd1 PLOTS=(DIAGNOSTICS);
  CLASS rep operator supplier day;
  MODEL strength = rep operator day supplier / SS3;
  RANDOM rep operator day / TEST ;
  MEANS supplier / BON ;
  MEANS rep operator day ;
TITLE 'Replicated Latin Square RLSD--1 Example';
RUN;
```

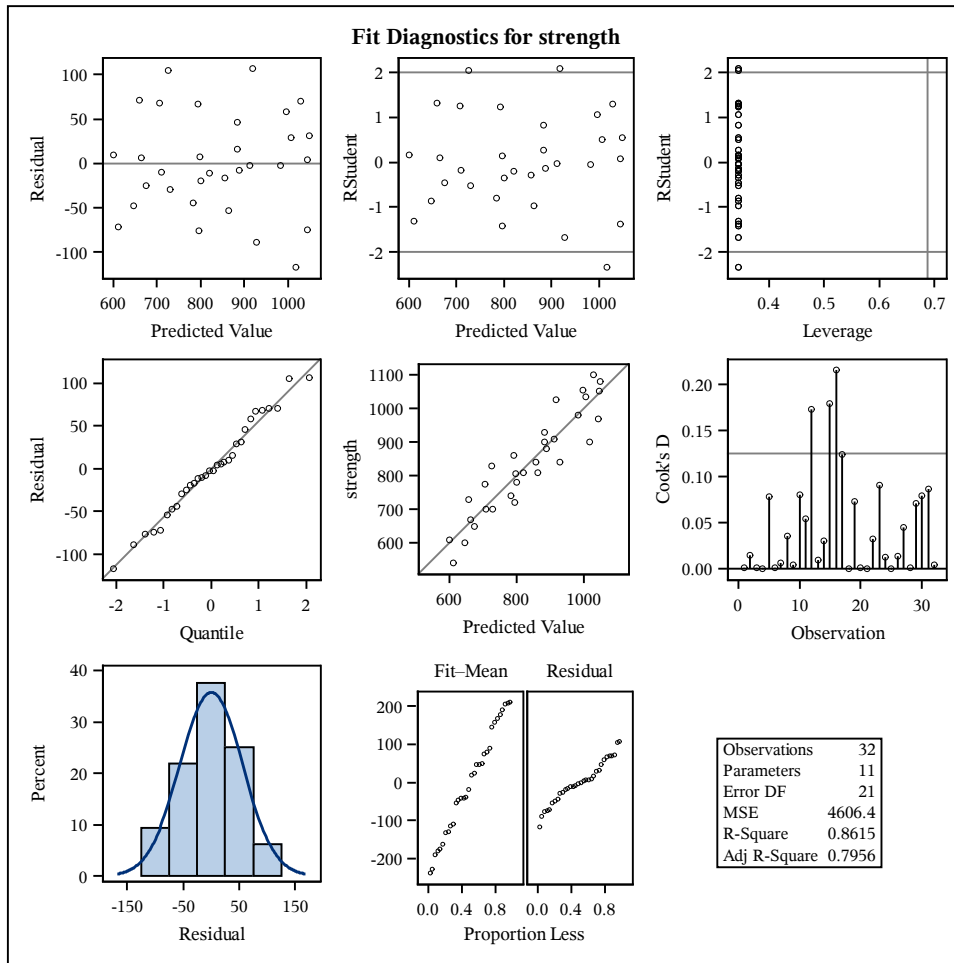
SAS output for RLSD-1 Example

Variable: strength

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|------------------------|----|----------------|-------------|---------|--------|
| Model | 10 | 601795.3125 | 60179.5313 | 13.06 | <.0001 |
| Error | 21 | 96735.1563 | 4606.4360 | | |
| Corrected Total | 31 | 698530.4688 | | | |

| R-Square | Coeff Var | Root MSE | strength Mean |
|----------|-----------|----------|---------------|
| 0.861516 | 8.096415 | 67.87073 | 838.2813 |

| Source | DF | Type III SS | Mean Square | F Value | Pr > F |
|-----------------|----|-------------|-------------|---------|--------|
| rep | 1 | 94.5313 | 94.5313 | 0.02 | 0.8875 |
| operator | 3 | 23852.3438 | 7950.7813 | 1.73 | 0.1923 |
| day | 3 | 4396.0938 | 1465.3646 | 0.32 | 0.8121 |
| supplier | 3 | 573452.3438 | 191150.7813 | 41.50 | <.0001 |



| Source | Type III Expected Mean Square |
|----------|--|
| rep | $\text{Var}(\text{Error}) + 16 \text{Var}(\text{rep})$ |
| operator | $\text{Var}(\text{Error}) + 8 \text{Var}(\text{operator})$ |
| day | $\text{Var}(\text{Error}) + 8 \text{Var}(\text{day})$ |
| supplier | $\text{Var}(\text{Error}) + Q(\text{supplier})$ |

The GLM Procedure
Tests of Hypotheses for Mixed Model Analysis of Variance

Variable: strength

| Source | DF | Type III SS | Mean Square | F Value | Pr > F |
|-------------------------|----|-------------|-------------|---------|--------|
| rep | 1 | 94.531250 | 94.531250 | 0.02 | 0.8875 |
| operator | 3 | 23852 | 7950.781250 | 1.73 | 0.1923 |
| day | 3 | 4396.093750 | 1465.364583 | 0.32 | 0.8121 |
| supplier | 3 | 573452 | 191151 | 41.50 | <.0001 |
| Error: MS(Error) | 21 | 96735 | 4606.436012 | | |

Bonferroni (Dunn) t Tests for strength

This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

| | |
|---------------------------------------|----------|
| Alpha | 0.05 |
| Error Degrees of Freedom | 21 |
| Error Mean Square | 4606.436 |
| Critical Value of t | 2.91209 |
| Minimum Significant Difference | 98.823 |

| Means with the same letter are not significantly different. | | | |
|--|-------------|----------|-----------------|
| Bon Grouping | Mean | N | supplier |
| A | 1021.25 | 8 | C |
| | | | |
| B | 891.88 | 8 | D |
| | | | |
| C | 780.63 | 8 | B |
| | | | |
| D | 659.38 | 8 | A |

| Level of rep | N | strength | |
|---------------------|----------|-----------------|----------------|
| | | Mean | Std Dev |
| 1 | 16 | 840.000000 | 170.029409 |
| 2 | 16 | 836.562500 | 132.862319 |

| Level of operator | N | strength | |
|--------------------------|----------|-----------------|----------------|
| | | Mean | Std Dev |
| 1 | 8 | 871.250000 | 133.490235 |
| 2 | 8 | 839.375000 | 177.209269 |
| 3 | 8 | 795.625000 | 174.671886 |
| 4 | 8 | 846.875000 | 129.033149 |

| Level of day | N | strength | |
|---------------------|----------|-----------------|----------------|
| | | Mean | Std Dev |
| 1 | 8 | 843.750000 | 148.028713 |
| 2 | 8 | 831.250000 | 186.121120 |
| 3 | 8 | 854.375000 | 140.622024 |
| 4 | 8 | 823.750000 | 151.108996 |

RLSD-3 EXAMPLE: A manufacturing firm investigated the breaking strengths of components made from raw materials purchased from 4 suppliers (A, B, C, D). Data was collected from 2 replicates of a 4 × 4 latin square design. The blocking factors were days and operators. Eight operators were used with four operators randomly assigned to each replicate. The two replicates were run over 8 days with the first 4 days assigned to replicate 1 and the second four days assigned to replicate 2.

SAS code for RLSD-3 Example

```
DM 'LOG; CLEAR; OUT; CLEAR;';
ODS GRAPHICS OFF;
ODS PRINTER PDF file='C:\COURSES\ST541\RLSD3.PDF';
OPTIONS NODATE NONUMBER;

*****;
*** REPLICATED LATIN SQUARE EXAMPLE RLSD-3 ***;
*****;
DATA rlsd3 ;
  DO rep      = 1 to 2;
  DO operator = 1 TO 4;
  DO day      = 1 TO 4;
  INPUT strength supplier $ @@; OUTPUT;
  END; END; END;
CARDS;
810 B 1080 C 700 A 910 D 1100 C 880 D 780 B 600 A
840 D 540 A 1055 C 830 B 650 A 740 B 1025 D 900 C
840 D 1050 C 775 A 805 B 670 A 930 D 720 B 1035 C
980 C 700 B 810 D 610 A 860 B 730 A 970 C 900 D

PROC GLM DATA=rlsd3 PLOTS=(ALL);
  CLASS rep operator supplier day;
  MODEL strength = rep operator(rep) day(rep) supplier / SS3;
  RANDOM rep operator(rep) day(rep) / TEST ;
  MEANS supplier / BON ;
  MEANS rep operator(rep) day(rep) ;
TITLE 'Replicated Latin Square RLSD--3 Example';
RUN;
```

SAS output for RLSD-3 Example

Variable: strength

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|------------------------|----|----------------|-------------|---------|--------|
| Model | 16 | 623343.7500 | 38958.9844 | 7.77 | 0.0001 |
| Error | 15 | 75186.7188 | 5012.4479 | | |
| Corrected Total | 31 | 698530.4688 | | | |

| R-Square | Coeff Var | Root MSE | strength Mean |
|----------|-----------|----------|---------------|
| 0.892364 | 8.445691 | 70.79864 | 838.2813 |

| Source | DF | Type III SS | Mean Square | F Value | Pr > F |
|----------------------|----|-------------|-------------|---------|--------|
| rep | 1 | 94.5312 | 94.5312 | 0.02 | 0.8926 |
| operator(rep) | 6 | 29904.6875 | 4984.1146 | 0.99 | 0.4638 |
| day(rep) | 6 | 19892.1875 | 3315.3646 | 0.66 | 0.6817 |
| supplier | 3 | 573452.3438 | 191150.7813 | 38.14 | <.0001 |

| Source | Type III Expected Mean Square |
|---------------|---|
| rep | Var(Error) + 4 Var(day(rep)) + 4 Var(operator(rep)) + 16 Var(rep) |
| operator(rep) | Var(Error) + 4 Var(operator(rep)) |
| day(rep) | Var(Error) + 4 Var(day(rep)) |
| supplier | Var(Error) + Q(supplier) |

Tests of Hypotheses for Mixed Model Analysis of Variance

Variable: strength

| Source | DF | Type III SS | Mean Square | F Value | Pr > F |
|---|--------|-------------|-------------|---------|--------|
| rep | 1 | 94.531250 | 94.531250 | 0.03 | 0.8864 |
| Error | 1.4129 | 4644.211027 | 3287.031250 | | |
| Error: MS(operator(rep)) + MS(day(rep)) - MS(Error) | | | | | |

| Source | DF | Type III SS | Mean Square | F Value | Pr > F |
|------------------|----|-------------|-------------|---------|--------|
| operator(rep) | 6 | 29905 | 4984.114583 | 0.99 | 0.4638 |
| day(rep) | 6 | 19892 | 3315.364583 | 0.66 | 0.6817 |
| supplier | 3 | 573452 | 191151 | 38.14 | <.0001 |
| Error: MS(Error) | 15 | 75187 | 5012.447917 | | |

Bonferroni (Dunn) t Tests for strength

This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

| | |
|--------------------------------|----------|
| Alpha | 0.05 |
| Error Degrees of Freedom | 15 |
| Error Mean Square | 5012.448 |
| Critical Value of t | 3.03628 |
| Minimum Significant Difference | 107.48 |

| Means with the same letter are not significantly different. | | | |
|---|---------|---|----------|
| Bon Grouping | Mean | N | supplier |
| A | 1021.25 | 8 | C |
| | | | |
| B | 891.88 | 8 | D |
| | | | |
| C | 780.63 | 8 | B |
| | | | |
| D | 659.38 | 8 | A |

| Level of rep | N | strength | |
|--------------|----|------------|------------|
| | | Mean | Std Dev |
| 1 | 16 | 840.000000 | 170.029409 |
| 2 | 16 | 836.562500 | 132.862319 |

| Level of operator | Level of rep | N | strength | |
|-------------------|--------------|---|------------|------------|
| | | | Mean | Std Dev |
| 1 | 1 | 4 | 875.000000 | 161.348484 |
| 2 | 1 | 4 | 840.000000 | 208.486610 |
| 3 | 1 | 4 | 816.250000 | 211.399109 |
| 4 | 1 | 4 | 828.750000 | 166.752061 |
| 1 | 2 | 4 | 867.500000 | 124.532459 |
| 2 | 2 | 4 | 838.750000 | 172.644867 |
| 3 | 2 | 4 | 775.000000 | 159.269164 |
| 4 | 2 | 4 | 865.000000 | 100.829890 |

| Level of day | Level of rep | N | strength | |
|--------------|--------------|---|------------|------------|
| | | | Mean | Std Dev |
| 1 | 1 | 4 | 850.000000 | 186.368810 |
| 2 | 1 | 4 | 810.000000 | 227.742545 |
| 3 | 1 | 4 | 890.000000 | 176.682389 |
| 4 | 1 | 4 | 810.000000 | 144.452991 |
| 1 | 2 | 4 | 837.500000 | 127.638813 |
| 2 | 2 | 4 | 852.500000 | 166.608323 |
| 3 | 2 | 4 | 818.750000 | 107.422453 |
| 4 | 2 | 4 | 837.500000 | 178.629038 |

RLSD-2 EXAMPLE: A study was performed to compare four baby food formula treatments. A 4×4 latin square design was replicated 4 times. The blocking factors were infants and weeks. (Actually, we have a simple repeated measures with infants receiving all 4 formula treatments.) A total of 16 infants were randomly assigned to the 4 replicates. The replicates were run over the same four weeks. The data is contained in the following tables (from *The Design and Analysis of Clinical Experiments* by J.L. Fleiss). If you temporarily ignore assumptions about the response, does this design meet the conditions to be a RLSD?

SAS code for RLSD-2 Example

```

DM 'LOG; CLEAR; OUT; CLEAR;';
ODS GRAPHICS OFF;
ODS PRINTER PDF file='C:\COURSES\ST541\RLSD2.PDF';
OPTIONS NODATE NONUMBER;

*****
*** REPLICATED LATIN SQUARE EXAMPLE RLSD-2 ***
*****
DATA rlsd2 ;
  DO square = 1 TO 4;
  DO week = 1 TO 4;
  DO _infant = 1 TO 4;
    infant = 4*(square-1) + _infant;
    INPUT diet formula @@; OUTPUT;
  END; END; END;

CARDS;
0.40 2 0.20 3 1.14 1 1.08 4 1.11 3 1.04 4 1.11 2 1.34 1
1.16 4 0.57 1 1.32 3 1.73 2 0.88 1 0.80 2 1.38 4 1.55 3
1.55 2 0.11 3 0.22 1 0.53 4 0.89 3 1.05 4 0.96 2 1.25 1
0.16 4 0.68 1 1.45 3 0.61 2 0.55 1 0.98 2 0.82 4 1.91 3
0.27 2 0.50 3 0.32 1 0.09 4 1.16 3 0.70 4 1.63 2 0.30 1
0.59 4 0.93 1 0.55 3 1.34 2 0.45 1 0.96 2 0.79 4 1.09 3
0.73 2 0.64 3 -0.03 1 1.05 4 1.21 3 1.38 4 1.04 2 1.11 1
1.21 4 0.82 1 0.57 3 1.00 2 0.77 1 0.79 2 0.55 4 0.50 3
;
PROC GLM DATA=rlsd2 PLOTS=(ALL);
  CLASS square infant formula week;
  MODEL diet = square infant(square) formula week / SS3;
  RANDOM square infant(square) week / TEST;
  MEANS formula / BON;
  MEANS square infant(square) week ;
TITLE 'Replicated Latin Square RLSD2--2 Example';
RUN;

```


Results of a study performed as four replicated Latin squares comparing four formulas fed to newborn infants

| Infant | Square 1 | | | | Mean |
|---------|----------|---------|---------|---------|----------|
| | 1 | 2 | 3 | 4 | |
| 1 | 0.40(2) | 1.11(3) | 1.16(4) | 0.88(1) | 0.8875 |
| 2 | 0.20(3) | 1.04(4) | 0.57(1) | 0.80(2) | 0.6525 |
| 3 | 1.14(1) | 1.11(2) | 1.32(3) | 1.38(4) | 1.2375 |
| 4 | 1.08(4) | 1.34(1) | 1.73(2) | 1.55(3) | 1.4250 |
| Mean | 0.7050 | 1.1500 | 1.1950 | 1.1525 | 1.050625 |
| Formula | 1 | 2 | 3 | 4 | |
| Mean | 0.9825 | 1.0100 | 1.0450 | 1.1650 | |

| Infant | Square 2 | | | | Mean |
|---------|----------|---------|---------|---------|--------|
| | 1 | 2 | 3 | 4 | |
| 1 | 1.55(2) | 0.89(3) | 0.16(4) | 0.55(1) | 0.7875 |
| 2 | 0.11(3) | 1.05(4) | 0.68(1) | 0.98(2) | 0.7050 |
| 3 | 0.22(1) | 0.96(2) | 1.45(3) | 0.82(4) | 0.8625 |
| 4 | 0.53(4) | 1.25(1) | 0.61(2) | 1.91(3) | 1.0750 |
| Mean | 0.6025 | 1.0375 | 0.7250 | 1.0650 | 0.8575 |
| Formula | 1 | 2 | 3 | 4 | |
| Mean | 0.6750 | 1.0250 | 1.0900 | 0.6400 | |

Square 3

| Infant | Week | | | | Mean |
|---------|---------|---------|---------|---------|----------|
| | 1 | 2 | 3 | 4 | |
| 1 | 0.27(2) | 1.16(3) | 0.59(4) | 0.45(1) | 0.6175 |
| 2 | 0.50(3) | 0.70(4) | 0.93(1) | 0.96(2) | 0.7725 |
| 3 | 0.32(1) | 1.63(2) | 0.55(3) | 0.79(4) | 0.8225 |
| 4 | 0.09(4) | 0.30(1) | 1.34(2) | 1.09(3) | 0.7050 |
| Mean | 0.2950 | 0.9475 | 0.8525 | 0.8225 | 0.729375 |
| Formula | 1 | 2 | 3 | 4 | |
| Mean | 0.5000 | 1.0500 | 0.8250 | 0.5425 | |

Square 4

| Infant | Week | | | | Mean |
|--------------|----------|----------|----------|----------|-----------|
| | 1 | 2 | 3 | 4 | |
| 1 | 0.73(2) | 1.21(3) | 1.21(4) | 0.77(1) | 0.9800 |
| 2 | 0.64(3) | 1.38(4) | 0.82(1) | 0.79(2) | 0.9075 |
| 3 | -0.03(1) | 1.04(2) | 0.57(3) | 0.55(4) | 0.5325 |
| 4 | 1.05(4) | 1.11(1) | 1.00(2) | 0.50(3) | 0.9150 |
| Mean | 0.5975 | 1.1850 | 0.9000 | 0.6525 | 0.83375 |
| Formula | 1 | 2 | 3 | 4 | |
| Mean | 0.6675 | 0.8900 | 0.7300 | 1.0475 | |
| Week | 1 | 2 | 3 | 4 | Mean |
| Overall mean | 0.550000 | 1.080000 | 0.918125 | 0.923125 | 0.8678125 |
| Formula | 1 | 2 | 3 | 4 | Mean |
| Overall mean | 0.70625 | 0.99375 | 0.92250 | 0.84875 | 0.8678125 |

Replicated Latin Square RLSD2--2 Example

The GLM Procedure

| Class Level Information | | |
|-------------------------|--------|--|
| Class | Levels | Values |
| square | 4 | 1 2 3 4 |
| infant | 16 | 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 |
| formula | 4 | 1 2 3 4 |
| week | 4 | 1 2 3 4 |

| | |
|-----------------------------|----|
| Number of Observations Read | 64 |
| Number of Observations Used | 64 |

Variable: diet

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|-----------------|----|----------------|-------------|---------|--------|
| Model | 21 | 6.34661875 | 0.30221994 | 2.32 | 0.0102 |
| Error | 42 | 5.48047500 | 0.13048750 | | |
| Corrected Total | 63 | 11.82709375 | | | |

| R-Square | Coeff Var | Root MSE | diet Mean |
|----------|-----------|----------|-----------|
| 0.536617 | 41.62541 | 0.361231 | 0.867813 |

| Source | DF | Type III SS | Mean Square | F Value | Pr > F |
|----------------|----|-------------|-------------|---------|--------|
| square | 3 | 0.86163125 | 0.28721042 | 2.20 | 0.1021 |
| infant(square) | 12 | 2.33401250 | 0.19450104 | 1.49 | 0.1662 |
| formula | 3 | 0.72506875 | 0.24168958 | 1.85 | 0.1524 |
| week | 3 | 2.42590625 | 0.80863542 | 6.20 | 0.0014 |

| Source | Type III Expected Mean Square |
|----------------|---|
| square | Var(Error) + 4 Var(infant(square)) + 16 Var(square) |
| infant(square) | Var(Error) + 4 Var(infant(square)) |
| formula | Var(Error) + Q(formula) |
| week | Var(Error) + 16 Var(week) |

The GLM Procedure
Tests of Hypotheses for Mixed Model Analysis of Variance

Variable: diet

| Source | DF | Type III SS | Mean Square | F Value | Pr > F |
|----------------------------------|----|-------------|-------------|---------|--------|
| square | 3 | 0.861631 | 0.287210 | 1.48 | 0.2704 |
| Error | 12 | 2.334013 | 0.194501 | | |
| Error: MS(infant(square)) | | | | | |

| Source | DF | Type III SS | Mean Square | F Value | Pr > F |
|------------------|----|-------------|-------------|---------|--------|
| infant(square) | 12 | 2.334013 | 0.194501 | 1.49 | 0.1662 |
| formula | 3 | 0.725069 | 0.241690 | 1.85 | 0.1524 |
| week | 3 | 2.425906 | 0.808635 | 6.20 | 0.0014 |
| Error: MS(Error) | 42 | 5.480475 | 0.130487 | | |

Bonferroni (Dunn) t Tests for diet

This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

| | |
|--------------------------------|----------|
| Alpha | 0.05 |
| Error Degrees of Freedom | 42 |
| Error Mean Square | 0.130487 |
| Critical Value of t | 2.76902 |
| Minimum Significant Difference | 0.3536 |

| Means with the same letter are not significantly different. | | | |
|--|--------|----|---------|
| Bon Grouping | Mean | N | formula |
| A | 0.9938 | 16 | 2 |
| A | | | |
| A | 0.9225 | 16 | 3 |
| A | | | |
| A | 0.8488 | 16 | 4 |
| A | | | |
| A | 0.7063 | 16 | 1 |

| Level of square | N | diet | |
|-----------------|----|------------|------------|
| | | Mean | Std Dev |
| 1 | 16 | 1.05062500 | 0.40607830 |
| 2 | 16 | 0.85750000 | 0.51014377 |
| 3 | 16 | 0.72937500 | 0.42829069 |
| 4 | 16 | 0.83375000 | 0.34993095 |

| Level of infant | Level of square | N | diet | |
|-----------------|-----------------|---|------------|------------|
| | | | Mean | Std Dev |
| 1 | 1 | 4 | 0.88750000 | 0.34711910 |
| 2 | 1 | 4 | 0.65250000 | 0.35752622 |
| 3 | 1 | 4 | 1.23750000 | 0.13275918 |
| 4 | 1 | 4 | 1.42500000 | 0.27982137 |
| 5 | 2 | 4 | 0.78750000 | 0.58937113 |
| 6 | 2 | 4 | 0.70500000 | 0.42790186 |
| 7 | 2 | 4 | 0.86250000 | 0.50638424 |
| 8 | 2 | 4 | 1.07500000 | 0.64319515 |
| 9 | 3 | 4 | 0.61750000 | 0.38465352 |
| 10 | 3 | 4 | 0.77250000 | 0.21561926 |
| 11 | 3 | 4 | 0.82250000 | 0.57151115 |
| 12 | 3 | 4 | 0.70500000 | 0.60379356 |
| 13 | 4 | 4 | 0.98000000 | 0.26608269 |
| 14 | 4 | 4 | 0.90750000 | 0.32469216 |
| 15 | 4 | 4 | 0.53250000 | 0.43805441 |
| 16 | 4 | 4 | 0.91500000 | 0.28029746 |

| Level of week | N | diet | |
|---------------|----|------------|------------|
| | | Mean | Std Dev |
| 1 | 16 | 0.55000000 | 0.45086583 |
| 2 | 16 | 1.08000000 | 0.29691750 |
| 3 | 16 | 0.91812500 | 0.42060621 |
| 4 | 16 | 0.92312500 | 0.39799026 |