# § 2.1 Solutions

3. Planes in row picture <u>have changed</u>.
   the vectors in column picture <u>have changed</u>.
   the coefficient matrix has <u>changed</u>.
   the solution <u>remains the same</u>.

5. Because $x, y, z$ satisfy the 1st 2 equations, then they also <u>intersect</u>.

the line L of solutions is found by solving the $2 \times 3$ system

$$\boxed{\begin{aligned} x + y + z &= 2 \\ x + 2y + z &= 3 \end{aligned}}$$

Subtracting them shows that $\boxed{y = 1}$. the new system is

$$\boxed{x + z = 1}$$

which defines all solutions to the <u>system</u>.

Three solutions are:

$$\boxed{\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}.}$$

Later, we will write the infinite number of solutions as

$$\boxed{\begin{pmatrix} 1-z \\ 1 \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + z \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}.}$$

7. No solution because there is <u>no interaction</u> to all 3 planes

9. (a) $\begin{pmatrix} 18 \\ 5 \\ 0 \end{pmatrix}$

(b) $\begin{pmatrix} 3 \\ 4 \\ 5 \\ 5 \end{pmatrix}$

10. The linear combination of columns is $\begin{pmatrix} 18 \\ 5 \\ 0 \end{pmatrix}$.

In an $n \times n$ matrix, there are $n^2$ multiplications!

12. (a) $\begin{pmatrix} z \\ y \\ x \end{pmatrix}$ (b) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ (c) $\begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix}$.

13. $n$ components to get $m$ components

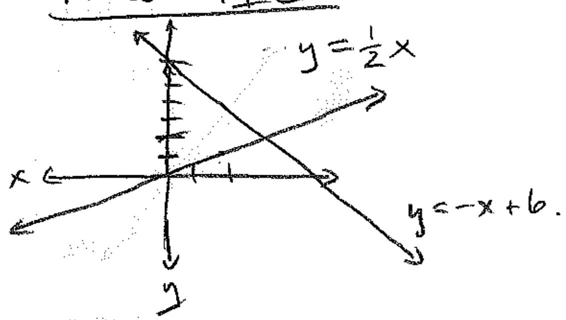
15. (a) $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(b) $I = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

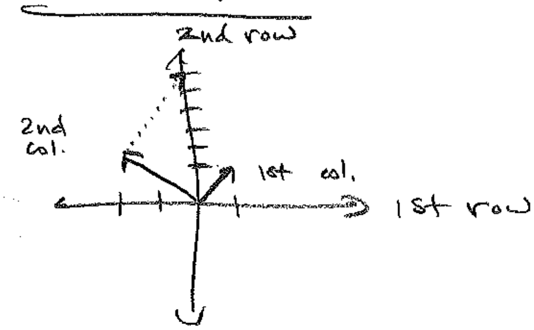
16. In general, the $2 \times 2$ matrix that rotates by $\theta$ is $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.

17. $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$, $Q = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

26. Row PIc:




Column PIc.




27. Row PIc has 2 planes in 3D

Col. PIC is in 2-D.

The solutions lie on a line.