§ 2.3 Solutions.

16. (A) $\left.\begin{aligned} 2x - y &= 0 \\ x + y &= 33 \end{aligned}\right\}$ original System.

$$\begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 33 \end{pmatrix}$$ $\Big\}$ Matrix System $A\underline{x} = \underline{b}$

The ref of augmented matrix is:

$$\begin{pmatrix} 2 & -1 & 0 \\ 0 & 3/2 & 33 \end{pmatrix}$$

So Backsubstitution shows ~~that~~ the solution is:

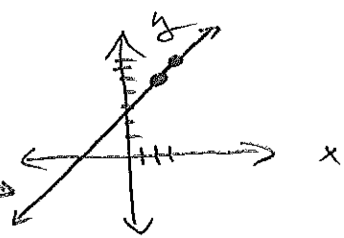
$$\underline{x} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 \\ 22 \end{pmatrix}.$$

(B) the equations to solve:

$$\begin{aligned} 2m + c &= 5 \\ 3m + c &= 7 \end{aligned}$$

Matrix system: $\begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} m \\ c \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$

the solution is $\underline{x} = \begin{pmatrix} m \\ c \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$



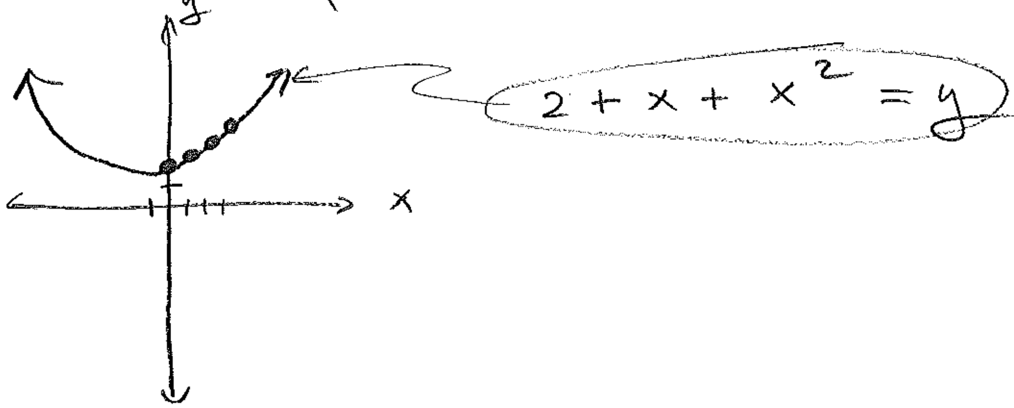
$y = 2x + 1$

17. the system of equations to solve is:

$$a + b + c = 4$$
$$a + 2b + 4c = 8$$
$$a + 3b + 9c = 14.$$

Matrix system: $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \\ 14 \end{pmatrix}.$

Solution is: $\underline{x} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}.$



24. GE applied to the augmented matrix yields:

$$\begin{pmatrix} 2 & 3 & 1 \\ 0 & -5 & 15 \end{pmatrix} \Longleftrightarrow \begin{pmatrix} 2 & 3 \\ 0 & -5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 15 \end{pmatrix}.$$

By back substitution, the solution is

$$\underline{x} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

(25.) The augmented matrix is

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 3 & 4 & 2 \\ 3 & 5 & 7 & 6 \end{bmatrix}$$

$R_2 \leftarrow -2R_1 + R_2$

$R_3 \leftarrow -3R_1 + R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -1 & -2 & 0 \\ 0 & -1 & -2 & 3 \end{bmatrix}$$

Now to eliminate below the <u>2nd pivot</u> = (-1).

$R_3 \leftarrow -1R_2 + R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -1 & -2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}.$$

The final equation says that $0 = 3$!

thus, there are <u>NO</u> solutions.

Changing the 6 in the RHS to a 3 yields an infinite # of solutions.