§ 2.4 Solutions

1. $\underset{5\times3}{\underset{\sim}{B}}\ \underset{3\times5}{\underset{\sim}{A}}$ is 5×5 matrix of 3's.

$\underset{3\times5}{\underset{\sim}{A}}\ \underset{5\times3}{\underset{\sim}{B}}$ is 3×3 matrix of 5's.

$\underset{3\times5}{\underset{\sim}{A}}\ \underset{5\times3}{\underset{\sim}{B}}\ \underset{3\times1}{\underset{\sim}{D}}$ is 3×1 vector of 15's.

$\underset{3\times1}{\underset{\sim}{D}}\ \underset{5\times3}{\underset{\sim}{B}}\ \underset{3\times5}{\underset{\sim}{A}}$ is impossible to compute

DANGER WILL ROBINSON! CANNOT MULTIPLY

$\underset{3\times5}{\underset{\sim}{A}}\ (\underset{5\times3}{\underset{\sim}{B}} + \underset{5\times1}{\underset{\sim}{C}})$ is IMPOSSIBLE.

CANNOT ADD

3. $AB = \begin{bmatrix} 0 & 7 \\ 0 & 7 \end{bmatrix}$, $AC = \begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix}$

So $AB + AC = \begin{bmatrix} 3 & 8 \\ 6 & 9 \end{bmatrix}$.

$B + C = \begin{bmatrix} 3 & 3 \\ 0 & 1 \end{bmatrix}$

So $A(B+C) = \begin{bmatrix} 3 & 8 \\ 6 & 9 \end{bmatrix}$.

this confirms the distributive property, that

$$A(B+C) = AB + AC.$$

6. $(A+B)(A+B) = A^2 + AB + BA + B^2$

14. (A) TRUE.

(B) FALSE, $\underset{m\times n}{\underset{\sim}{A}}\ \underset{n\times m}{\underset{\sim}{B}}$ is $m\times m$; and $\underset{n\times m}{\underset{\sim}{B}}\ \underset{m\times n}{\underset{\sim}{A}}$ is $n\times n$

(C) TRUE

(D) FALSE, $AB = B \Leftrightarrow (A - I)B = 0$

So $A = I$ is the only option when B is invertible. But when $B = 0$, $AB = B$ is also satisfied.

20. $$A^2 = \begin{bmatrix} 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$A^4 = \underset{4\times 4}{\underset{\sim}{0}}$ ← the zero matrix.

So $A\underset{\sim}{v} = \begin{pmatrix} 2y \\ 2z \\ 2t \\ 0 \end{pmatrix}$, $A^2\underline{v} = \begin{pmatrix} 4z \\ 4t \\ 0 \\ 0 \end{pmatrix}$, $A^3\underline{v} = \begin{pmatrix} 8t \\ 0 \\ 0 \\ 0 \end{pmatrix}$, $A^4\underline{v} = \underset{4\times 1}{0}$

zero vector

(21.) $A^n = A$ for any n!

$$AB = \begin{pmatrix} 1/2 & -1/2 \\ 1/2 & -1/2 \end{pmatrix}$$

$$(AB)^n = \underset{2\times 2}{\underset{\sim}{0}} \quad \text{for any } n > 1.$$

(26.)

$$\underset{3\times 2}{\underset{\sim}{A}}\,\underset{2\times 3}{\underset{\sim}{B}} = \begin{bmatrix} 3 & 3 & 0 \\ 10 & 14 & 4 \\ 7 & 8 & 1 \end{bmatrix}$$

(32.) If $X = \begin{bmatrix} | & | & | \\ \underset{\sim}{x}_1 & \underset{\sim}{x}_2 & \underset{\sim}{x}_3 \\ | & | & | \end{bmatrix}$, then

$$AX = \begin{bmatrix} | & | & | \\ A\underset{\sim}{x}_1 & Ax_2 & Ax_3 \\ | & | & | \end{bmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= I.$$

That is X is A^{-1}!