§ 2.5 Solutions

(2)
$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

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(3.) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1/2 \\ -1/5 \end{bmatrix}$ and $\begin{bmatrix} t \\ z \end{bmatrix} = \begin{bmatrix} -1/5 \\ 1/10 \end{bmatrix}$

$$\Rightarrow A^{-1} = \begin{bmatrix} 1/2 & -1/5 \\ -1/5 & 1/10 \end{bmatrix}.$$

(4.) When solving
$$\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Augmented matrix is
$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & 0 \end{bmatrix} \xrightarrow{R_2 \leftarrow -3R_1 + R_2} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & -3 \end{bmatrix}$$

$\therefore$ $0 = -3$, ABSURD!

So A^{-1} cannot exist bc applying GJE to

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 3 & 6 & 0 & 1 \end{bmatrix} \text{ yields } \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 0 & -3 & 1 \end{bmatrix}$$

A ZERO instead of a 2nd pivot!

(5.) $U = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix}$

(6.) (A) Multiply both sides on the left by A^{-1}.

(B) A just adds up components in columns.

So e.g. $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1/2 & 1/4 \\ 1/2 & 3/4 \end{bmatrix}$

(10.) $A^{-1} = \begin{bmatrix} & 0\text{'s} & 1/4 & 1/5 \\ & 1/3 & & 0\text{'s} \\ 1/2 & & & \end{bmatrix}$, $B^{-1} = \begin{bmatrix} 3 & -2 & 0 & 0 \\ -4 & 3 & 0 & 0 \\ 0 & 0 & 6 & -5 \\ 0 & 0 & -7 & 6 \end{bmatrix}$

(11.) E.g. $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$.

(13.) $M^{-1} = (ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

$\Rightarrow B^{-1} = CM^{-1}A$.

(19.) $a = 2/5$, $b = 1/5$.

(23.) $A^{-1} = \begin{bmatrix} 3/4 & -1/2 & 1/4 \\ -1/2 & 1 & -1/2 \\ 1/4 & -1/2 & 3/4 \end{bmatrix}$.

(25.) $A^{-1} = \begin{bmatrix} 3/4 & -1/4 & -1/4 \\ -1/4 & 3/4 & -1/4 \\ -1/4 & -1/4 & 3/4 \end{bmatrix}$; B^{-1} does not exist.

(28.)

$A^{-1} = \begin{bmatrix} -1/2 & 1/2 \\ 1/2 & 0 \end{bmatrix}$.

29) A. TRUE, because a row of 0's yields a zero pivot, det(matrix) = product of pivots

$= 0$

B. FALSE, $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ is not invertible.

C. TRUE, by properties of inverses

$$(A^{-1})^{-1} = A, \quad (A^2)^{-1} = (A^{-1})^2.$$

30.

$$\left[\begin{array}{ccc|ccc} 2 & c & c & 1 & 0 & 0 \\ c & c & c & 0 & 1 & 0 \\ 8 & 7 & c & 0 & 0 & 1 \end{array}\right]$$

$R_2 \leftarrow c/_2 R_1 + R_2$
$R_3 \leftarrow -4R_1 + R_3$

$$\left[\begin{array}{ccc|ccc} 2 & c & c & 1 & 0 & 0 \\ 0 & c - c^2/_2 & c - c^2/_2 & -c/_2 & 1 & 0 \\ 0 & 7-4c & -3c & -4 & 0 & 1 \end{array}\right].$$

So if $c - c^2/_2 = 0$, there is a missing pivot in the 2nd row: this occurs when $\boxed{c = 0 \text{ or } 2}$

If $c \neq 0$ and $c \neq 2$, then the next step is to apply $\frac{7-4c}{c - c^2/_2} R_2 + R_3 \rightarrow R_3$. The 3rd pivot is $(7-4c) + -3c = 7 - 7c$. This pivot is missing if $\boxed{c = 1}$

32) $$\underset{4\times4}{A^{-1}} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \underset{5\times5}{A^{-1}} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

So $\underline{x} = \begin{pmatrix} 2 \\ 2 \\ 2 \\ 1 \end{pmatrix}$ is solution of $\underline{A}\,\underline{x} = \underset{4\times1}{\underline{1}}$