§ 3.1 Solutions

#1 (1) $\underline{x}+\underline{y} \neq \underline{y}+\underline{x}$ (commutativity of vector addition)

because if $\underline{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\underline{y} = \begin{bmatrix} 10 \\ 20 \end{bmatrix}$, then

using goofy definition $\underline{x}+\underline{y} = \begin{bmatrix} x_1+y_2 \\ x_2+y_1 \end{bmatrix}$:

$\underline{x}+\underline{y} = \begin{bmatrix} 21 \\ 12 \end{bmatrix}$ but $\underline{y}+\underline{x} = \begin{bmatrix} 12 \\ 21 \end{bmatrix}$.

(2) $\underline{x}+(\underline{y}+\underline{z}) \neq (\underline{x}+\underline{y})+\underline{z}$ either!

(associativity of vector addition)

(8) $(a+b)\underline{x} \neq a\underline{x}+b\underline{x}$!

(associativity of scalar multiplication).

#2 Property (5) is violated.

#4 $\underline{0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $\frac{1}{2}A = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$, $-A = \begin{bmatrix} -2 & 2 \\ -2 & 2 \end{bmatrix}$

the smallest vector space containing A is the space $= \{cA : \text{for } c \in \mathbb{R}\}$

#6 $h(x) = 3x^2 - 4(5x)$

POINT: The set of vectors (i.e. functions) generated by x^2 & $5x$ is a vector space.

#10 (A.) Yes because of closure under vector addition (A1)

$$\begin{pmatrix} c \\ c \\ A \end{pmatrix} + \begin{pmatrix} d \\ d \\ B \end{pmatrix} = \begin{bmatrix} c+d \\ c+d \\ A+B \end{bmatrix}$$ is in the same space

Closure under Scalar Mult:

$$c\begin{bmatrix} d \\ d \\ B \end{bmatrix} = \begin{bmatrix} cd \\ cd \\ cB \end{bmatrix}$$ is in the same space.

(B) No, you'll add 2 of these vectors and get $b_1 = 2$!

(C) No, eg. consider closure under (A1), vector addition, of $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

(D.) ABSOLUTELY.

(E.) Yes — check closure under vector addition (A1) and scalar multiplication (M1) to convince yourself.

(F.) Yes. Notice if the requirement was that $b_1 < b_2 < b_3$, then the answer would be No!

#12 This exercise shows that P is <u>NOT</u> a vector space.

#13 $x+y-2z=0$ defines vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ that do form a vector space. Check this by confirming closure under vector addition, A1 and closure under scalar multiplication, M1.

#14 (A) $\mathbb{R}^2$ subspaces are all of $\mathbb{R}^2$, lines that go through $\underline{0}$, or $\underline{0}$ itself.

(B) the space of all 2×2 matrices is equivalent to $\mathbb{R}^4$. But the space of all diagonal matrices is equivalent to $\mathbb{R}^2 \subset \mathbb{R}^4$.

#16 $\underline{\text{all of } \mathbb{R}^3}$ or $\underline{\text{just } P}$

↑ if L is independent of P

↑ if L is in the plane P.

#19 $C(A)$ is a line $= c\cdot\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ for all $c\in\mathbb{R}$.
$C(B)$ is a plane $= a\cdot\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + b\begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$ for all $a,b\in\mathbb{R}$
$C(C)$ is a line $= c\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ for all $c\in\mathbb{R}$.

(22.) $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ has det $= 1$, so is invertible,

so gives 1 solution to $A\underline{x} = \underline{b}$

for any $\underline{b} \in \mathbb{R}^3$.

In other words, $C(A) = \mathbb{R}^3$

$B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ has det $= 0$, so only has

solutions for $\underline{b} = a\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + b\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

because $C(B) = $ all l.c's of $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ has det $= 0$, so only has

solutions for $\underline{b} = a\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + b\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

because $C(C) = $ all l.c.'s of $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

(26.) $C(\underset{5\times 5}{\underline{A}}) = \mathbb{R}^5$ because $\underset{5\times5}{\underline{A}}\,\underset{5\times1}{\underline{x}} = \underset{5\times1}{\underline{b}}$

can be solved for any $\underline{b}$ because is

invertible. If $C(A) \neq \mathbb{R}^5$, then we

could find $\underline{b}^* \in \mathbb{R}^5$ not in $C(A)$, in which

case $A\underline{x} = \underline{b}^*$ would have no solutions,

contradicting that A is invertible.

#27 A. FALSE, because $\underline{0} \in C(A)$.

B. TRUE.

C. TRUE

D. FALSE, let $A = I$.

#28 $C\left(\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\right)$ does not contain $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. —

try to solve $A\underline{x} = \underline{b}$ for

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

to prove it!

The matrices $\underbrace{\begin{bmatrix} 1 & 1 & 1 \\ \underline{x} & \underline{0} & \underline{0} \\ 1 & 1 & 1 \end{bmatrix}}_{3\times 3}$ or $\underbrace{\begin{bmatrix} 1 & 1 & 1 \\ \underline{x} & \underline{x} & \underline{x} \\ 1 & 1 & 1 \end{bmatrix}}_{3\times 3}$

have column spaces = a line.

#29 $\mathbb{R}^9$ — otherwise, you would be able to pick a $\underline{b} \in \mathbb{R}^9$ that was not in $C(A)$, in which case $\underbrace{A}_{9\times 12} \underbrace{\underline{x}}_{12\times 1} = \underbrace{\underline{b}}_{9\times 1}$ would <u>not</u> be solvable.