Solutions to §3.2

(1) $R_2 \leftarrow -R_1 + R_2$ $\qquad\qquad$ $R_3 \leftarrow -R_2 + R_3$

(a)
$$\begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix} \qquad \text{ref}(A) = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(b) After $R_3 \leftarrow -2R_2 + R_3$

$$\text{ref}(B) = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 4 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

(5) $$\text{ref}(A) = \begin{bmatrix} -1 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{ref}(B) = \begin{bmatrix} -1 & 3 & 5 \\ 0 & 0 & -3 \end{bmatrix}$$

(7) (A) Solving $A\underline{x} = \underline{0}$, with $\underline{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$,

Let y + z be free, thus

$$-x + 3y + 5z = 0$$

$\Rightarrow$ solution is

$$\underline{x} = \begin{bmatrix} 3y + 5z \\ y \\ z \end{bmatrix} = y \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$$

Thus $N(A) = \text{span}\left(\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix} \right)$.

(B) Solving $B\underline{x} = \underline{0}$ with $\underline{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$,

let $\boxed{y \text{ be free}}$, ref(B) shows that $z = 0$ and

$$-x + 3y + 5z = 0$$

$$\Rightarrow -x + 3y = 0$$

$$\Rightarrow \underline{x} = \begin{bmatrix} 3y + 5z \\ y \\ 0 \end{bmatrix} = y\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + z\begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

thus $N(A) = \text{Span}\left(\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix}\right)$.

9 (A) FALSE. Consider $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

for which y is free.

(B) TRUE, because if A is invertible, then $A\underline{x} = \underline{b}$ has only 1 solution for all RHSs $\underline{b}$.

$\Rightarrow N(A) = \{\underline{0}\}$

$\Rightarrow$ No free variables

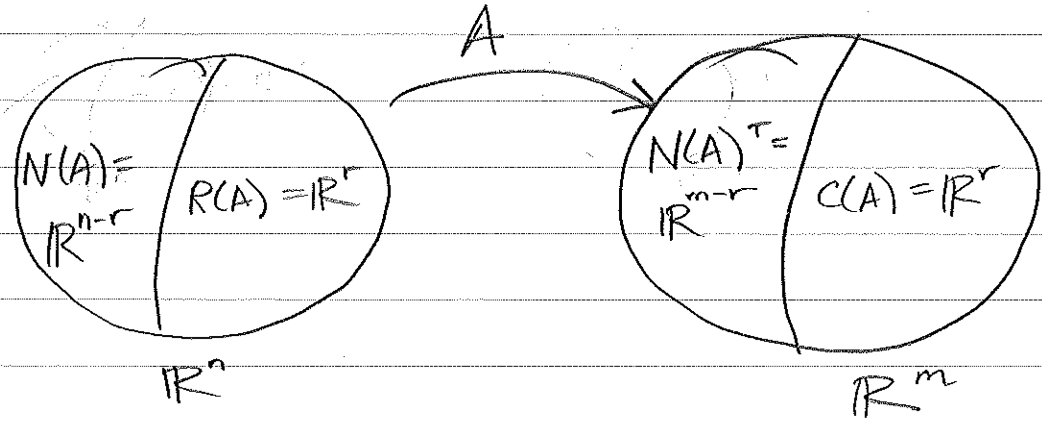
(C) TRUE

(D) TRUE

(13) $\begin{bmatrix} \times & \times & \times & 0 & \times \\ \times & \times & \times & 0 & \times \\ \times & \times & \times & 0 & \times \end{bmatrix} \Rightarrow \boxed{x_4 \text{ is free}}$

3×5

(15) The blob picture is:



$N(A) = \{\underline{0}\}$ when $r = n$.
$C(A) = \mathbb{R}^m$ when $r = m$.

(17) $A = [1 \ -3 \ -1]$. You can choose any variables to be free. If we choose $\boxed{y \text{ and } z \text{ as free}}$, then the solution is

$$\underline{x} = \begin{bmatrix} 3y + z \\ y \\ z \end{bmatrix} = y \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore N(A) = \text{span}\left(\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right).$$

(18) $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix} + y\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + z\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$

(29) $\operatorname{rref}(\underset{3\times 3}{\tilde{A}}) = \underset{3\times 3}{\tilde{I}}$ most likely.

$$\operatorname{rref}(\underset{4\times 3}{\tilde{A}}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$ most likely.