Solutions §5.1

#2. $\det(\tfrac{1}{2}\underset{3\times3}{\underbrace{A}}) = \det\left(\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{bmatrix} \cdot A\right)$

$$= \det\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{bmatrix} \cdot \det(A)$$

$$= \left(\tfrac{1}{2}\right)^3 (-1)$$

$$= -1/8.$$

$$\det(-\underset{3\times3}{\underbrace{A}}) = \det\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \cdot \det(A)$$

$$= (-1)^3(-1)$$

$$= +1.$$

$$\det(A^2) = \det(A)\det(A) = (-1)(-1)$$

$$= +1.$$

$$\det(A^{-1}) = \frac{1}{\det(A)} = -1.$$

POINT: $\boxed{\det(\underset{1\times1}{\underbrace{c}}\,\underset{n\times n}{\underbrace{A}}) = c^n \det(A).}$

#3. A. FALSE, determinants do not distribute across sums:

$$\det\left(\underset{2\times 2}{\underset{\sim}{I}} + \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\right) = \det\begin{pmatrix} 2 & 2 \\ 3 & 5 \end{pmatrix} = 4.$$

but $1 + \det\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = -1.$

In general, $\boxed{\det(A+B) \neq \det(A) + \det B}$!

B. TRUE

$\det(ABC) = \det(A)\det(B)\det(C)$

follows easily from the property that

$\det(DE) = \det(D)\det(E)$.

C. FALSE

$$\det\left(4\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\right) = \det\begin{pmatrix} 4 & 8 \\ 12 & 16 \end{pmatrix} = 64 - 96 = -32$$

which is not equal to

$$4 \cdot \det\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = 4 \cdot (-2) = -8.$$

The correct formula is

$$\det\left(4\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\right) = 4^2 \det\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

(see point in #2 of §5.1).

D. FALSE because determinants DO NOT distribute across a sum!

Consider $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. then

$$AB - BA = \begin{bmatrix} 0 & 0 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 0 & 2 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 3 & -4 \end{bmatrix}$$

which has $\det(AB - BA) = \det\begin{bmatrix} 0 & -2 \\ 3 & -4 \end{bmatrix} = 6$

#12 Using point in #2 in §5.1,

$$\det(A^{-1}) = \det\left(\left(\frac{1}{ad-bc}\right)\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\right)$$

$$= \left(\frac{1}{ad-bc}\right)^2 \det\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \left(\frac{1}{ad-bc}\right)^2 (ad-bc)$$

$$= \frac{1}{ad-bc}$$

$$= 1/\det(A).$$

§5.1 Solutions (cont'd)

9. $|A| = 1$, $|B| = 2$, $|C| = 0$.

13. det(First matrix) = 1, det(2nd matrix) = 3

14. det(1st) = 36, det(2nd) = 5.

15. det(1st) = 0, det(2nd) = $1 - 2t^2 + t^4$

19. det(1st U) = 6

det(1st U^{-1}) = $1/6$

det(1st U^2) = 36

det(2nd U) = ad

det(2nd U^{-1}) = $1/ad$

det(2nd U^2) = $(ad)^2$

22. $\det(A) = 3$, $\det(A^{-1}) = 1/3$

$\det(A - \lambda I) = 3 - 4\lambda + \lambda^2$

$\therefore \lambda = 1$ and $3 \Rightarrow \det(A - \lambda I) = 0$.