Exam 1

MATH 221-02

February 20, 2015 Chapters 1, 2, and 5

1. (28 pts) Circle **T** or **F** below indicating whether the following statements are True or False. You do <u>not</u> need to justify your answers for this problem.

(a) **T** **F** $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

(b) **T** **F** Let $\boldsymbol{u} = [1 \;\; 2 \;\; 3]$ and $\boldsymbol{v} = [3 \;\; 6 \;\; 9]$. All linear combinations $a\boldsymbol{u} + b\,\boldsymbol{v}$ (a and b are scalars) form a plane in three dimensions.

(c) **T** **F** Let $\boldsymbol{u}$ and $\boldsymbol{v}$ be $n \times 1$ vectors. If $\boldsymbol{u} \cdot \boldsymbol{v} = 1$ then the angle between $\boldsymbol{u}$ and $\boldsymbol{v}$ is between 0 and 90 degrees.

(d) **T** **F** If $\boldsymbol{u}$ and $\boldsymbol{v}$ are $n \times 1$ vectors, then $\boldsymbol{u} \cdot \boldsymbol{v} \leq ||\boldsymbol{u}||\;||\boldsymbol{v}||$.

(e) **T** **F** If $\boldsymbol{u}$ and $\boldsymbol{v}$ are $n \times 1$ vectors, then $||\boldsymbol{u}|| + ||\boldsymbol{v}|| \leq \|\boldsymbol{u} + \boldsymbol{v}\|$.

(f) **T** **F** Let S be a 3×3 matrix with columns $\boldsymbol{s}_1$, $\boldsymbol{s}_2$, and $\boldsymbol{s}_3$. Then the linear combination $\boldsymbol{s}_1 - \boldsymbol{s}_2 + 5\boldsymbol{s}_3$ is equal to the matrix vector product $S \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix}$.

(g) **T** **F** $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(h) **T** **F** You find that an $n \times n$ matrix A has no solutions to $A\boldsymbol{x} = \boldsymbol{b}$ for one RHS $\boldsymbol{b}$. Then $A\boldsymbol{x} = \boldsymbol{c}$ has no solutions for any other RHS $\boldsymbol{c}$.

(i) **T** **F** $(A^{-1}B^{-1})^{-1} = AB$.

(j) **T** **F** If A and B are $n \times n$ matrices then $(A - B)^2 = A^2 - 2AB + B^2$.

(k) **T** **F** If A and B are $n \times n$ matrices, and $\det(A) \neq 0$ and $\det(B) \neq 0$, then the matrix $C = AB$ is invertible.

(l) **T** **F** If $\det(A) = 5$, then $\det(10A) = 50$.

(m) **T** **F** If A is an $m \times n$ matrix, then $A^T A$ is symmetric.

(n) **T** **F** If A is invertible and symmetric, then A^{-1} is symmetric.

2. (8 pts) Prove that $\boldsymbol{u} \cdot \boldsymbol{v} = ||\boldsymbol{u}||||\boldsymbol{v}|| \cos(\theta)$ where θ is the angle between $\boldsymbol{u}$ and $\boldsymbol{v}$. Justify each step in your proof.

3. (14 pts) In each of the following, circle an appropriate statement from the options <u>*statement1/statement2*</u>. You do not need to justify your answers.

Assume that A is $n \times n$ and that the matrix system $A\boldsymbol{x} = \boldsymbol{b}$ has one solution.

(a) A is <u>singular/nonsingular</u>.
(b) A is <u>invertible/(not invertible)</u>.
(c) $A\boldsymbol{x} = \boldsymbol{b}$ <u>has/(does not have)</u> non-zero homogeneous (nullspace) solutions.
(d) $\det(A)$ <u>is/(is not)</u> equal to zero.
(e) $\boldsymbol{b}$ <u>is/(is not)</u> a linear combination of the columns of A.
(f) The columns of A are linearly <u>dependent/independent</u>.
(g) When performing Gauss-Elimination to put the augmented matrix into row echelon form (assuming there are no row switches), there <u>are/(are not)</u> n non-zero pivot(s).

4. (6 pts) Fill in the blanks with the <u>number of solutions</u>. You do not need to justify your answers.

(a) The system $A\boldsymbol{x} = \boldsymbol{b}$ with augmented matrix $\left(\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 0 & 10 & 1 & 1 \\ 0 & 10 & 1 & 1 \end{array}\right)$ has ________ solution(s).

(b) The system $A\boldsymbol{x} = \boldsymbol{b}$ with augmented matrix $\left(\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 7 & 0 & 0 \end{array}\right)$ has ________ solution(s).

5. (8 pts) The system $A\boldsymbol{x} = \boldsymbol{b}$ has augmented matrix $\left(\begin{array}{ccc|c} 1 & 6 & 3 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$.

(a) Find the solution(s) to the system $A\boldsymbol{x} = \boldsymbol{b}$. Identify any free variables if there are any. SHOW YOUR WORK!

(b) Write the solution(s) as a <u>particular solution</u> plus a <u>homogeneous (or nullspace) solution</u>, carefully identifying each.

6. Xavier is x years old. Yolande is y years old. Xavier is twice as old as Yolande; and their ages add to 33.

   (a) (4 pts) Write out the linear system $A\boldsymbol{x} = \boldsymbol{b}$ that must be solved in order to find x and y. Carefully identify A, $\boldsymbol{x}$ and $\boldsymbol{b}$ in your answer.

   (b) (8 pts) Give the augmented matrix then use Gauss Elimination to put the augmented matrix into row echelon form and solve for x and y. SHOW YOUR WORK!

   (c) (4 pts) Write the RHS $\boldsymbol{b}$ as a linear combination of the columns of the coefficient matrix A.

7. Let $A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$.

(a) (8 pts) Without actually trying to find the inverse of A, show that it is invertible. SHOW YOUR WORK and explain why your work proves invertibility.

(b) (12 pts) Find A^{-1}. SHOW YOUR WORK!