QUIZ 11 KEY

Let $u_{k+1} = u_k - 5v_k$
$v_{k+1} = 2u_k + 3v_k$

#1 Find characteristic polynomial. (1pt)

Rewrite the system as

$$\underline{u}_{k+1} = \begin{bmatrix} u_{k+1} \\ v_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & -5 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} u_k \\ v_k \end{bmatrix} = A\underline{u}_k$$

the characteristic polynomial of A is

$$\det(A - \lambda I) = \begin{vmatrix} 1-\lambda & -5 \\ 2 & 3-\lambda \end{vmatrix} = (1-\lambda)(3-\lambda) + 10$$

$$\boxed{\lambda^2 - 4\lambda + 13}$$

#2 FIND EIGENPAIRS (3pts)

Setting to 0, $\lambda = \dfrac{4 \pm \sqrt{16 - 4(1)(13)}}{2} = \dfrac{4 \pm \sqrt{-36}}{2}$

$$\boxed{\lambda = 2 \pm 3i}$$

these are the eigenvalues. To get eigenvector for $\lambda = 2 + 3i$, solve

$$(A - (2+3i)I)\underline{s} = \underline{0}$$

The augmented matrix for this system is:

$$\begin{bmatrix} -1-3i & -5 & 0 \\ 2 & 1-3i & 0 \end{bmatrix}$$

$$R_2 \leftarrow \frac{-2}{(-1-3i)} R_1 + R_2$$

$$\begin{bmatrix} -1-3i & -5 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Let $\boxed{s_2 \text{ be free}}$, then $s_1 = 5s_2 / -1-3i$

$$= \frac{5s_2(-1+3i)}{(-1-3i)(-1+3i)} = \frac{(-5+15i)s_2}{10}$$

$$\therefore\ s_1 = \left(-\frac{1}{2} + \frac{3}{2}i\right)s_2$$

$$\Rightarrow \underline{s} = s_2\begin{bmatrix} -\frac{1}{2} + \frac{3}{2}i \\ 1 \end{bmatrix} = s_2\left[\begin{bmatrix} -1/2 \\ 1 \end{bmatrix} + i\begin{bmatrix} 3/2 \\ 0 \end{bmatrix}\right].$$

$$\boxed{\begin{array}{l}\therefore \text{ eigenpairs of } A \text{ are} \\ \left(2+3i, \begin{bmatrix} -1/2 \\ 1 \end{bmatrix} + i\begin{bmatrix} 3/2 \\ 0 \end{bmatrix}\right) \text{ \& } \left(2-3i, \begin{bmatrix} -1/2 \\ 1 \end{bmatrix} - i\begin{bmatrix} 3/2 \\ 0 \end{bmatrix}\right).\end{array}}$$

Scaling by $s_2 = i$ get eigenvectors:

$$\begin{bmatrix} -3/2 \\ 0 \end{bmatrix} \pm i\begin{bmatrix} -1/2 \\ 1 \end{bmatrix}$$

Scaling by $s_2 = 2$, get:

$$\begin{bmatrix} -1 \\ 2 \end{bmatrix} \pm i\begin{bmatrix} 3 \\ 0 \end{bmatrix}.$$

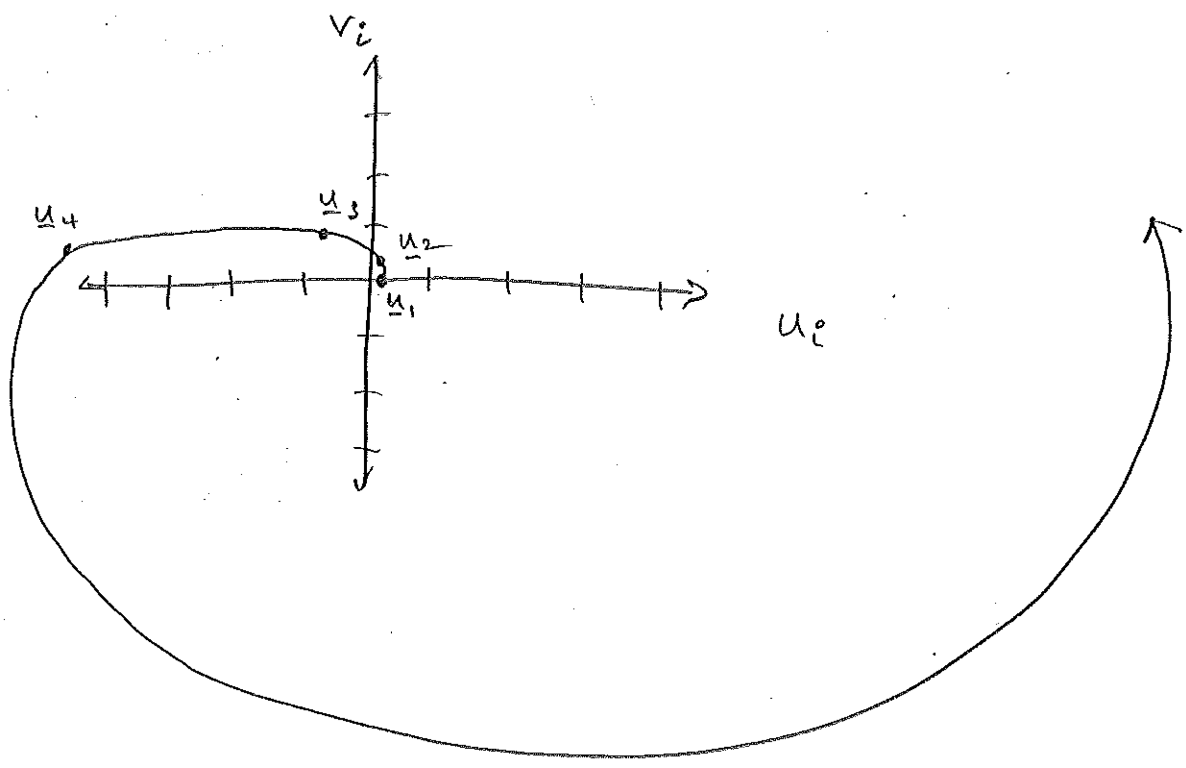
#3. If $\underline{u}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, plot $\underline{u}_2, \underline{u}_3, \underline{u}_4$.

$$\underline{u}_2 = A\underline{u}_1 = \begin{bmatrix} 1 & -5 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

$$\underline{u}_3 = A\underline{u}_2 = \begin{bmatrix} 1 & -5 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -9 \\ 8 \end{bmatrix}$$

$$\underline{u}_4 = A\underline{u}_3 = \begin{bmatrix} 1 & -5 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} -9 \\ 8 \end{bmatrix} = \begin{bmatrix} -49 \\ 6 \end{bmatrix}$$

The plot is



#4 Predict behavior for any $\underline{u}_1$. Because longest $|\text{eigenvalue}| = |2 \pm 3i| = \sqrt{13} > 1$, then long term behavior increases in magnitude. Because these eigenvalues are complex, the long term behavior oscillates as a spiral in the above plot.