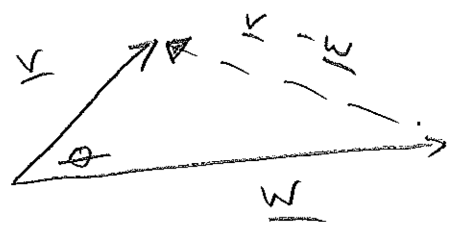
Quiz 2 solutions

(1) Prove that $\underline{v} \cdot \underline{w} = \|\underline{v}\| \times \|\underline{w}\| \cos\theta$.

pf: The law of cosines shows that



$$\boxed{\|\underline{v} - \underline{w}\|^2 = \|\underline{v}\|^2 + \|\underline{w}\|^2 - 2\|\underline{v}\| \times \|w\| \times \cos\theta}$$

the LHS can be expanded:

$$\|\underline{v} - \underline{w}\|^2 = (\underline{v} - \underline{w}) \cdot (\underline{v} - \underline{w})$$

$$= \underline{v} \cdot \underline{v} - \underline{v}\,\underline{w} - \underline{w}\,\underline{v} + \underline{w} \cdot \underline{w}.$$

$$\boxed{\|\underline{v} - \underline{w}\|^2 = \underline{v} \cdot \underline{v} - 2\underline{v} \cdot \underline{w} + \underline{w} \cdot \underline{w}.}$$

Setting the equations in the 2 boxes equal ~~to~~ each other shows:

$$\|\underline{v}\|^2 + \|\underline{v}\|^2 - 2\|\underline{v}\| \times \|\underline{w}\| \cos\theta = v \cdot v - 2v \cdot w + w \cdot w$$

$$= \|v\|^2 - 2v \cdot w + \|\underline{w}\|^2$$

since $v \cdot v = \|\underline{v}\|^2$ and $w \cdot w = \|w\|^2$.

Cancelling terms from both sides shows: $\|\underline{v}\| \|\underline{w}\| \cos\theta = v \cdot w$.