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Quiz 3
MATH 221-2
February 4, 2015   Sections 2.1-2.2

Consider the system of linear equations (# 5 in section 2.2)

$$3x + 2y = 10$$
$$6x + 4y = 20$$

1. Write out the augmented matrix for this system.

$$\begin{bmatrix} 3 & 2 & 10 \\ 6 & 4 & 20 \end{bmatrix}$$

2. Apply Gauss Elimination to put the augmented matrix into row echelon form. Show your work.

$$R_2 \leftarrow -2R_1 + R_2$$

$$\begin{bmatrix} 3 & 2 & 10 \\ 0 & 0 & 0 \end{bmatrix}$$

$$0 = 0 \:!$$

Let $\boxed{y \text{ be free}}$

Then $3x = 10 - 2y$

$x = \dfrac{10}{3} - \dfrac{2}{3}y$
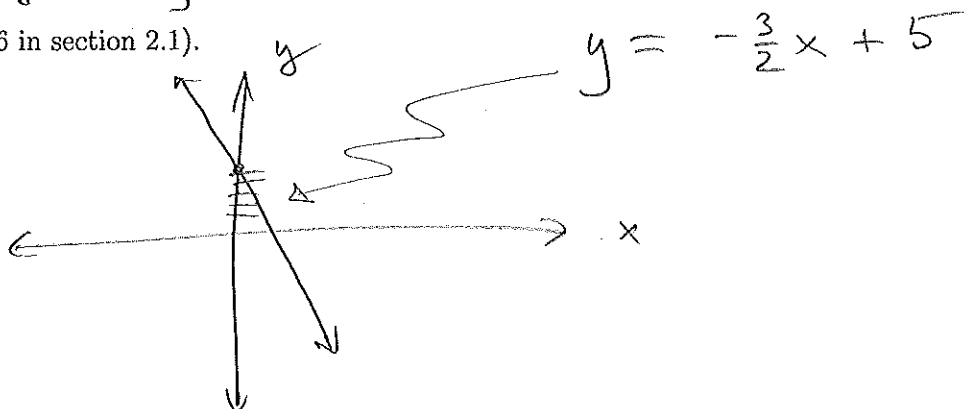
3. How many solutions are there? If there is/are solution(s), give it. If not, explain.

There are an infinite number solutions of the form

$$\underline{x} = \begin{bmatrix} \frac{10}{3} - \frac{2}{3}y \\ y \end{bmatrix}.$$

4. Draw the "row picture" (#26 in section 2.1).

$$y = -\frac{3}{2}x + 5$$

5. If there is/are solution(s), write it as a particular solution plus a homogeneous (or nullspace) solution $\vec{x} = \vec{x}_p + \vec{x}_n$, clearly indicating each.

$$\underline{x} = \begin{bmatrix} 10/3 - 2/3\,y \\ y \end{bmatrix} = \underbrace{\begin{bmatrix} \tfrac{10}{3} \\ 0 \end{bmatrix}}_{\underline{x}_p} + y \underbrace{\begin{bmatrix} -2/3 \\ 1 \end{bmatrix}}_{\underline{x}_n}$$

6. Based on your answers to 3 and 5, write the RHS vector $b = \begin{pmatrix} 10 \\ 20 \end{pmatrix}$ as a linear combination of the columns of the coefficient matrix A.

$$\begin{bmatrix} 10 \\ 20 \end{bmatrix} = x \begin{bmatrix} 3 \\ 6 \end{bmatrix} + y \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \tfrac{10}{3} \begin{bmatrix} 3 \\ 6 \end{bmatrix} + 0 \begin{bmatrix} 2 \\ 4 \end{bmatrix}.$$

7. Draw the "column picture" (#26 in section 2.1).
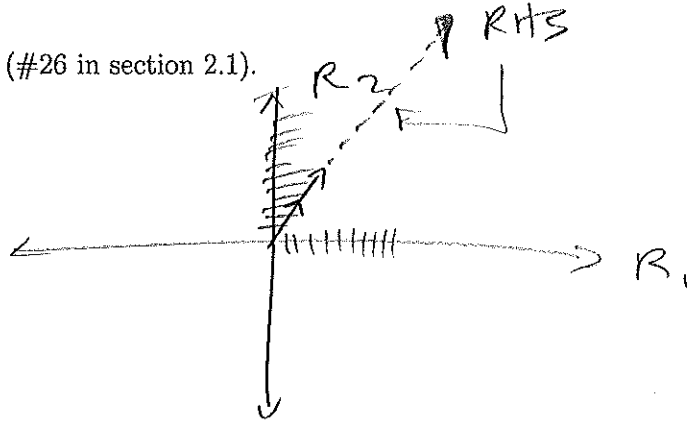
8. Describe geometrically (line or plane) all linear combinations of the columns of the coefficient matrix A (#1 in section 1).

LINE.

9. Is the coefficient matrix A ⟨singular and non-invertible⟩ or non-singular and invertible? Explain why your answer is correct.

Because the columns are dependent; also because there is a 0 instead of pivot in ref!