KEY

/6

Quiz 4

MATH 221-2

February 13, 2015 Section 2.5.

Answer the questions on both sides of this sheet

1. Show that $\begin{pmatrix} 0 & 1 & 1 \\ 2 & 0 & 2 \\ 0 & 1 & 0 \end{pmatrix}$ is invertible by either directly calculating a determinant, and/or by finding the pivots.

The Laplace cofactor expansion along the 1st row is:

$$0 + (-1)(1)\begin{vmatrix} 2 & 2 \\ 0 & 0 \end{vmatrix} + 1\begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix}$$

$-1(0) + 2 = \boxed{2} \neq 0$ so A is invertible.

Also, note that GE elimination fails because there is a 0 instead of the 1st pivot. After switching 1st and 2nd rows, get:

$$\begin{bmatrix} 2 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \xrightarrow{R_3 \leftarrow -R_2 + R_3} \begin{bmatrix} 2 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

which has 3 pivots, so original A is invertible.

2. Find the inverse of $\begin{pmatrix} 0 & 1 & 1 \\ 2 & 0 & 2 \\ 0 & 1 & 0 \end{pmatrix}$.

Starting with the augmented matrix

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 2 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Since 0 instead of pivot switch 1st & 2nd rows.

$$\begin{bmatrix} 2 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_3 \leftarrow -R_2 + R_3$

$$\begin{bmatrix} 2 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & 1 \end{bmatrix}$$

$R_1 \leftarrow R_1 + 2R_3$ $\quad R_2 \leftarrow R_2 + R_3$

$$\begin{bmatrix} 2 & 0 & 0 & -2 & 1 & 2 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & -1 & 0 & 1 \end{bmatrix}$$

$R_2 \leftarrow \frac{1}{2} R_2$

$R_3 \leftarrow -R_3$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & \frac{1}{2} & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & -1 \end{bmatrix}$$

Since $\begin{bmatrix} 0 & 1 & 1 \\ 2 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & \frac{1}{2} & 1 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Then $\boxed{A^{-1} = \begin{bmatrix} -1 & \frac{1}{2} & 1 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix}}$.