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Quiz 5

MATH 221-02

March 4, 2015 Chapter 3.

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$ (from problem #21b in section 3.4).

(2 pts) 1. Solve the equation $A\mathbf{x} = \mathbf{b}$ for $\mathbf{b} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$. SHOW YOUR WORK!

the augmented matrix is:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 1 & -1 & 1 & 4 \end{array}\right]$$

$R_2 \leftarrow -R_1 + R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & -2 & 0 & 0 \end{array}\right]$$

Let $\boxed{z \text{ be free}}$. the second row says: $-2y = 0$

$\boxed{y = 0}$

So $x = 4 - y - z = 4 - 0 - z = \boxed{4 - z = x}$

(1/2 pt) 2. How many solutions are there to $A\mathbf{x} = \mathbf{b}$?

An infinite number because $z \in \mathbb{R}$ is free!

(1 pt) 3. Write the solution as a particular plus a null solution, $\mathbf{x}_p + \mathbf{x}_n$.

The solution is $= \underline{x}_p + \underline{x}_n$

$$\underline{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4-z \\ 0 \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix} + z \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

(1 pt) 4. If possible, write the RHS $\mathbf{b} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$ as a linear combination of the columns of A? If not possible, explain why.

$$4\begin{bmatrix} 1 \\ 1 \end{bmatrix} + 0\begin{bmatrix} 1 \\ -1 \end{bmatrix} + 0\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$ because $\underline{x}_p = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$.

($1\frac{1}{2}$ pt) 5. Based on your answer to #4, explain whether or not $\mathbf{b}$ is in $C(A)$.

Since $\underline{b}$ is a linear combination of the columns of A, then, by definition, $\underline{b} \in C(A)$!

(1 pt) 6. Find the dimensionality of the domain of $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$. Explain why your answer is correct.

Because $\underset{2\times 3}{\underset{\sim}{A}} \; \underset{3\times 1}{\underset{\sim}{x}} = \underset{2\times 1}{\underset{\sim}{b}}$

and domain contains all $\underline{x}$'s, then $\boxed{\text{domain} = \mathbb{R}^3}$

(1 pt) 7. Find the dimensionality of the range or column space of $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$, $C(A)$. Show or explain why your answer is correct.

Only the 1st & 2nd columns are independent. So $C(A)$ is generated by l.c.'s. $a\begin{bmatrix} 1 \\ 1 \end{bmatrix} + b\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ for $a, b \in \mathbb{R}$. $\therefore$ $\boxed{C(A) = \mathbb{R}^2}$

(2 pt) 8. For $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$, show that the range (or column space) $C(A)$ is a vector space. SHOW YOUR WORK!

We need to prove (A1) & (M1). To prove (A1) (closure wrt vector addition)

Let $\underbrace{\underline{u}, \underline{v} \in C(A)}$ and see if $\underline{u} + \underline{v} \in C(A)$.

$\Downarrow$

$\underline{u} = a\begin{bmatrix} 1 \\ 1 \end{bmatrix} + b\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\underline{v} = c\begin{bmatrix} 1 \\ 1 \end{bmatrix} + d\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

So $\underline{u} + \underline{v} = \left[a\begin{bmatrix} 1 \\ 1 \end{bmatrix} + b\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right] + \left(c\begin{bmatrix} 1 \\ 1 \end{bmatrix} + d\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = (a+c)\begin{bmatrix} 1 \\ 1 \end{bmatrix} + (b+d)\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$=$ l.c. of $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ & $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

So $\underline{u} + \underline{v} \in C(A)$ by def.

To prove (M1) (closure wrt scalar mult), note $e\underline{u} = e\left(a\begin{bmatrix} 1 \\ 1 \end{bmatrix} + b\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = ea\begin{bmatrix} 1 \\ 1 \end{bmatrix} + eb\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

a. l.c. of $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ & $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

So $e\underline{u} \in C(A)$!