

KEY

Quiz 6 MATH 221-02

March 20, 2015

Chapter 3.

7

$$\text{Let } A = \begin{pmatrix} 1 & 3 & 0 & 0 \\ 2 & 6 & 1 & 0 \end{pmatrix}.$$

(2 pts) 1. Find a basis for $C(A)$. SHOW YOUR WORK!

Applying GE to A

$$R_1 \leftarrow -2R_1 + R_2$$

$$\begin{pmatrix} \textcircled{1} & 3 & 0 & 0 \\ 0 & 0 & \textcircled{1} & 0 \end{pmatrix}$$

Pivots in the 1st & 3rd columns show that

Basis for $C(A)$ is

$$\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$$

(1 pt) 2. Find a basis for $R(A)$. SHOW YOUR WORK!

From #1, pivots in 1st & 2nd rows show that

Basis for $R(A)$ is

$$\left\{ \begin{bmatrix} 1 \\ 3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \\ 1 \\ 0 \end{bmatrix} \right\}$$

(2 pts) 3. Find a basis for $N(A)$. SHOW YOUR WORK!

Solving for $A\underline{x} = \underline{0}$ gives augmented matrix

$$\begin{bmatrix} 1 & 3 & 0 & 0 & 0 \\ 2 & 6 & 1 & 0 & 0 \end{bmatrix}$$

From #1, the ref of augmented matrix is

$$\begin{bmatrix} \textcircled{1} & 3 & 0 & 0 & 0 \\ 0 & 0 & \textcircled{1} & 0 & 0 \end{bmatrix}$$

If $\underline{x} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$ then let $\boxed{b \text{ \& } d \text{ be free}}$

Augmented matrix $\Rightarrow \boxed{c = 0}$ and $\boxed{a = -3b}$

So $\underline{x} = \begin{bmatrix} -3b \\ b \\ 0 \\ d \end{bmatrix} = b \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + d \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \text{Basis of } N(A) \text{ is } \left\{ \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

(2 pts) 4. Write out the "blob diagram" that clearly shows where the domain of A , $C(A)$, $R(A)$, $N(A^T)$, and $N(A)$ reside, and indicates the dimensionality of each of these 5 vector spaces.

