Take Home Quiz 7

MATH 221-02

DUE: Friday, March 27, 2015

Sections 4.1-4.2.

Print out this quiz; write your answers on it; turn it in at the beginning of class on Friday.

1. Let
$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix}$$
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(a) Write out the "blob diagram" that clearly shows where the domain of A, C(A), R(A), $N(A^T)$, and N(A) reside, and indicate the dimensionality of each of these 5 vector spaces.

- (b) When solving Ax = b for any RHS $b \in \Re^3$, use the Fredholm Alternative to explain how many solutions you expect for get. *Hint: there are two possibilities.*
- (c) Find the 3×3 projection matrix that projects 3×1 vectors x onto C(A). SHOW YOUR WORK!

(d) Use your answer from #1c to project the vector $x = \begin{bmatrix} 5\\ 2\\ -1 \end{bmatrix}$ onto C(A). SHOW YOUR WORK!

- 2. Let $A = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix}$.
 - (a) Find a basis for N(A). SHOW YOUR WORK!

- (b) Write out the "blob diagram" that clearly shows where the domain of A, C(A), R(A), $N(A^T)$, and N(A) reside, and indicate the dimensionality of each of these 5 vector spaces.
- (c) When solving Ax = b for any RHS $b \in \Re^3$, use the Fredholm Alternative to argue how many solutions you expect for get. *Hint: there is only one possibility.*
- (d) Try to apply the method described in class to find the 2×2 projection matrix that projects 2×1 vectors x onto C(A). You will find that that the method fails. At which point does the method fail? SHOW YOUR WORK!

(e) What is the difference in the 4 fundamental spaces for the A matrix in #1c (where you could find the projection matrix) and the A matrix in #2d that could explain why you could not find the projection matrix in #2d?