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Quiz 8

MATH 221-02

DUE: Monday, April 13, 2015 Section 6.1, 6.2.

(Section 6.1, problem #2) Let $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$.

(2 pts) 1. Find the eigenvalues of A. Show your work.

Set $\det(A - \lambda I) = 0$

$$\begin{vmatrix} 1-\lambda & 4 \\ 2 & 3-\lambda \end{vmatrix} = (1-\lambda)(3-\lambda) - 8 = -5 - 4\lambda + \lambda^2$$

$$= (\lambda - 5)(\lambda + 1) = 0$$

$\therefore \lambda = -1, 5$

(2 pts) 2. Find the eigenvector corresponding to the eigenvalue of $\lambda = 5$. Show your work.

Solve: $(A - 5I)\underline{s} = \underline{0}$

$$\begin{bmatrix} -4 & 4 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Augmented Matrix:

$$\begin{bmatrix} -4 & 4 & 0 \\ 2 & -2 & 0 \end{bmatrix} \xrightarrow{R_2 \leftarrow \frac{1}{2}R_1 + R_2} \begin{bmatrix} -4 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Let $\boxed{s_2 \text{ be free}}$ Then $s_1 = {-4s_2}/{-4} = s_2$

$$\therefore \underline{s} = \begin{bmatrix} s_2 \\ s_2 \end{bmatrix} = s_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

(1 pt) 3. The eigenvector corresponding to $\lambda = -1$ is $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$. Together with your answer to #2, organize your eigenvectors into a matrix S, and organize your eigenvectors as a diagonal matrix Λ.

$$S = \begin{bmatrix} -2 & 1 \\ 1 & 1 \end{bmatrix}, \quad \Lambda = \begin{bmatrix} -1 & 0 \\ 0 & 5 \end{bmatrix}$$

(1 pt) 4. Can A be written as the eigendecomposition $A = S\Lambda S^{-1}$? Explain why or why not.

Yes, because there are 2 linearly independent eigenvectors $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, so S is invertible.

(1 pt) 5. Can all vectors in $\Re^2$ be written as a linear combination of the eigenvectors of A? Explain why or why not.

Yes! The 2 eigenvectors form a basis for $\mathbb{R}^2$. In other words, any $b \in \mathbb{R}^2$ can be written as

$$b = a\begin{bmatrix} -2 \\ 1 \end{bmatrix} + b\begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

(2 pts) 6. Draw the "blob diagram". Clearly show the eigenvectors of A in the left blob, and show how A stretches, squishes, flips or rotates $\Re^2$ in the right blob. What about the eigenvalues tells you <u>why</u> you get the stretches, squishes, flips or rotates?